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A Comparative Look at Sunspot Cycles

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Scientific and Technical Information Branch

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TABLE OF CONTENTS

		Page
I.	INTRODUCTION	1
II.	APPROACH	1
	A. Sunspot Number and Smoothed Sunspot Number	1
III.	RESULTS	6
	 A. Linear Regression Equations for Selected Measures of Solar Activity B. Parametric Values Versus SCN C. Mean Parametric Values Based on Selected Groupings of SCN D. Mean R_Z and R	6 13 25 25 32 42
IV.	DISCUSSION	53
	A. General Remarks Concerning Sunspot Cycles	53 53
V.	CONCLUSIONS	71
REFERI	ENCES	72
APPENI	DIX A	73
APPENI	DIX B	110
APPENI	DIX C	124
APPENI	DIX D	138
APPENI	DIX E	152
APPENI	DIX F	166

LIST OF ILLUSTRATIONS

Figure	Title	Page
1.	Schematic sunspot cycle showing selected parameters based on smoothed sunspot number versus time	4
2.	Schematic sunspot cycle showing selected parameters based on rate of change of smoothed sunspot number versus time	5
3.	Monthly mean sunspot number derived by AAVSO versus provisional monthly mean sunspot number derived by Zurich Observatory	7
4.	Final monthly mean sunspot number versus provisional monthly mean sunspot number	8
5.	Monthly mean 2800-MHz radio flux corrected for bursts and adjusted to 1 AU versus provisional monthly mean sunspot number	9
6.	Smoothed sunspot number versus time for cycles 19, 20, and 21	10
7.	Smoothed sunspot number versus final monthly mean sunspot number	11
8.	Smoothed 2800-MHz radio flux corrected for bursts and adjusted to 1 AU versus monthly mean 2800-MHz radio flux corrected for bursts and adjusted to 1 AU	12
9.	Parametric values versus SCN (in terms of ordinary number value)	17
10.	Parametric values versus SCN (in terms of mean and standard deviation value)	20
11.	Histograms of parametric values (in units of 0.5 s)	23
12.	Sunspot cycle phase versus time (suggesting long- and short-period groupings)	24
13.	Mean monthly sunspot number (based on mean of cycles 8 through 20) versus time from cycle minimum occurrence	27
14.	Standard deviation of monthly mean sunspot values versus time from cycle minimum occurrence	27
15.	Smoothed sunspot number (based on mean of cycles 8 through 20) versus time from cycle minimum occurrence	28
16.	Standard deviation of smoothed sunspot values versus time from cycle minimum occurrence	29

LIST OF ILLUSTRATIONS (Concluded)

Figure	Title	Page
17.	Smoothed sunspot number for HIGH- and LOW-R _{MAX} cycles versus time from cycle minimum occurrence	30
18.	Smoothed sunspot number for LONG- and SHORT-period cycles versus time from cycle minimum occurrence	31
19.	Selected parameters versus SCN	36
20.	Descent slope versus ascent slope	39
21.	Descent period versus ascent period	39
22.	Maximum cycle smoothed sunspot number versus ascent period	40
23.	Maximum cycle smoothed sunspot number versus minimum cycle smoothed sunspot number	42
24.	Monthly mean sunspot number for cycle 21 compared to mean monthly sunspot number based on cycles 8 through 20	61
25.	Smoothed sunspot number for cycle 21 compared to mean smoothed sunspot number based on cycles 8 through 20	62
26.	Smoothed sunspot number for cycle 21 compared to mean smoothed sunspot number based on HIGH- and LOW- \bar{R}_{MAX} cycles	63
27.	Smoothed sunspot number for cycle 21 compared to mean smoothed sunspot number based on LONG- and SHORT-period cycles	64
28.	Q ₁₃ value versus time from cycle 21 minimum smoothed sunspot number occurrence	69
29.	Comparison of cycle 21 smoothed sunspot number (versus time) with computed ascent slope and projected descent slope	70

LIST OF TABLES

Table	Title	Page
1.	Linear Regression Coefficients for Selected Parameters Based on Comparison to Provisional Value of Monthly Mean Sunspot Number	13
2.	Summary of Selected Sunspot Cycle Parameters for SCN 8 through 21	14
3.	Quick-Reference Table for Determining Above (+) and Below (-) Parametric Mean Value Cycles	16
4.	Parametric Values for Selected Groupings of Cycles	26
5.	Comparison of Schematic and Mean Curve Parametric Values for Selected Parameters (Based on Cycles 8 through 20)	29
6.	Comparison of Schematic and Mean Curve Parametric Values for Selected Parameters (Based on HIGH- and LOW-R _{MAX} Cycle Groupings)	31
7.	Comparison of Schematic and Mean Curve Parametric Values for Selected Parameters (Based on LONG- and SHORT-PERIOD Cycle Groupings)	32
8.	Mean Smoothed Sunspot Number Values Versus t for Selected Cycle Groupings	33
9.	List of Selected Parametric Data for SCN 8 through 21	37
10.	Linear Regression Coefficients for Cycle Parameters Based on Known Selected Early Occurring Cycle Parameters	43
11.	Linear Regression Coefficients for Cycle Parameters Based on Known Sums of Monthly Mean Sunspot Number for the Time Intervals 12, 18 and 24 Months	49
12.	Linear Regression Coefficients for Selected Cycle Parameters Based on Known Selected Late Occurring Cycle Parameters.	52
13.	Comparison of Observed Values of Selected Cycle Parameters for Cycle 21 with Various Estimates Based on Known \bar{R}_{MIN} and $\frac{\Delta}{GPV}$ \bar{R}_{13} -Based Linear	
	Regression Equations and on Mean Values for HIGH- and Low- \bar{R}_{MAX} and LONG- and SHORT-PERIOD Cycle Groupings	56
14.	Comparison of Observed Values of Cycle Parameters for Cycle 21 with Mean Values of Selected Estimates	59
15.	Comparison of Observed Values of Cycle Parameters for Cycle 21 with Estimates Based on Sums of Mean Monthly Sunspot Number (12, 18 and 24 Months) Linear Regression Equations	60
16.	Comparison of Cycle 21 and Mean of Cycles 8 through 20 in Terms of	
	$R_{\mathbf{Z}}, Q_{\mathbf{Z}}, R_{13}$, and Q_{13}	66

LIST OF SELECTED ABBREVIATIONS

Abbreviation Meaning

R Relative sunspot number

R_A American sunspot number

R_Z Zurich monthly mean sunspot number

R_I International monthly mean sunspot number

R₁₃ Smoothed sunspot number

F₂₈₀₀ Mean monthly 2800-MHz Ottawa solar radio flux, adjusted to 1 AU and corrected

for bursts

F₁₃ Smoothed 2800-MHz Ottawa solar radio flux, adjusted to 1 AU and corrected for

bursts

R_{MIN} Smoothed sunspot number at cycle minimum; marks beginning of sunspot cycle

R_{MIN}(SCN+1) Smoothed sunspot number at end of sunspot cycle; marks beginning of next

sunspot cycle

R_{MAX} Smoothed sunspot number at cycle maximum

R_{CHM} Corrected-half-maximum smoothed sunspot number

R_{MEAN} Mean smoothed sunspot number for a cycle

ASC Ascent period; time in months measured from \overline{R}_{MIN} to \overline{R}_{MAX} same cycle

DES Descent period; time in months measured from \overline{R}_{MAX} to \overline{R}_{MIN} next cycle

ASC $_{CHM}$ \subseteq Corrected-half-maximum ascent period; time in months measured from \overline{R}_{MIN} to

 \overline{R}_{CHM}

 D_{CHM} Corrected-half-maximum duration; time in months measured from \overline{R}_{CHM} ascent side

to \overline{R}_{CHM} descent side of cycle; time in months when \overline{R}_{13} is greater than or equal to

 \overline{R}_{CHM}

 $\frac{\Delta}{GNV} \overline{R}_{13}$ Greatest negative value of $\Delta \overline{R}_{13}$

 t_{GNV} Time of occurrence of $\frac{\Delta}{GNV} \; \overline{R}_{13}$ measured from \overline{R}_{MIN}

t_{GPV} Time of occurrence of $\Delta _{GPV}^{\Delta}$ \overline{R}_{13} measured from \overline{R}_{MIN}

 \overline{R}_{13} (t_{GNV}) Smoothed sunspot number at t_{GNV}

 \overline{R}_{13} (t_{GPV}) Smoothed sunspot number at t_{GPV}

SCN Sunspot cycle number

NASA TECHNICAL PAPER

A COMPARATIVE LOOK AT SUNSPOT CYCLES

I. INTRODUCTION

Sunspots, dark regions observable in telescopic white-light photographic images of the Sun and occasionally with the naked eye, have long fascinated man. While earliest known reference to naked-eye sunspot observations date back many centuries (some 2000 to 2300 years), it has only been about 370 vears since the first telescopic observations were made by Fabricius, Galileo, Harriot and Scheiner. Variation with time in spottedness, although noted by ancient and early solar observers, was not recognized as being "periodic," or more accurately "cyclic," until Heinrich Schwabe announced his findings in 1843; even then, sunspot cyclicity - the somewhat regular increase and decrease in number of sunspots with time - was not generally accepted until about 1850, following the introduction by Rudolph Wolf (in 1848) of the now well-known "relative sunspot number R" as a measure of sunspot activity. (Maunder [1] was among the first to adopt the term "sunspot cycle" instead of the term "sunspot period" as had previously been used.) Schove [2], Bray and Loughhead [3], and Meadows [4] have provided very readable "historical introductions" to sunspots, and Newkirk and Frazier [5] have summarized more recent findings; see also Noyes [6], Nicholson [7], Wallenhorst [8] and Stahl [9]. (Ron Moore of NASA/Marshall Space Flight Center is presently preparing an observational review on sunspots for Annual Review of Astronomy and Astrophysics.) In retrospect, then, it is seen that the highest quality solar data, those systematic telescopic observations of the Sun employing the relative sunspot number R, date back a mere 130 years or so. (Eddy [10] has discussed limitations concerning the accuracy or "quality" of the historic sunspot record; i.e., sunspot numbers determined prior to about 1850.)

The purpose of this paper is fivefold: (i) to review and compare sunspot numbers, in particular those measured between November 1833 and June 1976, corresponding to sunspot cycle numbers (SCN) 8 through 20; (ii) to determine statistical properties (e.g., mean values, standard deviations and range) for specific sunspot number-related parameters; (iii) to obtain frequency distributions of these parameters; (iv) to derive linear regression equations between selected parameters; and (v) to use these parameters and equations to estimate equivalent parameters for cycle 21. (At the time of manuscript preparation, final sunspot numbers for cycle 21 are known only through December 1981.) Section II describes the approach used in this study and defines a number of cycle-related parameters. (For convenience, a list is provided at the beginning of this report, following the List of Tables, which defines a select number of cycle-related parameters often used throughout the text.) Section III gives results and Section IV discusses how these results can be used to estimate cycle parametric values for cycle 21. Section V states the conclusions. This report represents a continuation of previous work concerning sunspot cycles and solar activity associated with them [11-16].

II. APPROACH

A. Sunspot Number and Smoothed Sunspot Number

The Wolf relative sunspot number R is defined as

 $R = k(10g + f) \qquad , \tag{1}$

where f is the total number of sunspots observed regardless of size, g is the number of observed sunspot groups, and k a normalization parameter which varies from observatory to observatory to bring counts into agreement by accounting for telescope size, atmospheric opacity, etc. Modern determinations of sunspot number are made daily on a world-wide basis. These daily counts, based on equation (1), are then averaged together to yield monthly mean sunspot numbers. (Annual averages are sometimes employed but will not be used in this study.) Of historical importance are those measurements of sunspot number made until very recently by the Swiss Federal Observatory in Zurich, Switzerland and its two branch stations in Arosa and Lacarno. These Zurich monthly mean sunspot numbers are denoted R_Z. Beginning in December 1980, an international sunspot number, denoted R_I, came into use and replaced R_Z. It is computed, using equation (1), by the Sunspot Index Data Center in Brussels, Belgium.

Two other measures of sunspot activity have also been developed. These include the American relative sunspot number (whose monthly mean value is denoted R_A) and the Ottawa solar flux value at 2800 MHz, adjusted to 1 AU and corrected for bursts (denoted in this study as F_{2800}). R_A is computed, using equation (1), based on observations made by the Solar Division of the American Association of Variable Star Observers (AAVSO). F_{2800} is a direct measure of the Sun's 2800-MHz radio emission (10.7-cm wavelength radio emission), in solar flux units (where 1 sfu = 10^{-22} W m⁻² Hz⁻¹), adjusted to 1 astronomical unit (where 1 AU is approximately 1.5 x 10^8 km), made by the Algonquin Radio Observatory of the National Research Council of Canada near Ottawa. R_Z (now R_I) is provided as a "provisional" value, whereas R_A and F_{2800} are often reported as "final" values. Final values for R_Z are usually reported the first quarter of the year succeeding the year of interest. Final and provisional values differ only slightly. Values for R_A , R_Z , and/or F_{2800} appear in several easily accessible publications, chief of which are the Solar-Geophysical Data — Prompt Reports, Journal of Geophysical Research, and Sky and Telescope. (Waldmeier [17] has published sunspot data for 1610 to 1960 and Allen [18] gives additional sunspot information.)

While daily sunspot numbers are the primary data, more often monthly mean sunspot numbers are used for comparative studies. These monthly mean sunspot numbers, however, still show considerable variation, so a particular smoothing technique has come into use which reduces the month-to-month scatter yet appropriately shows the general trend and level of solar activity with time. The "smoothed sunspot number" or 13-month running mean is denoted here as \overline{R}_{13} and is defined as

$$\overline{R}_{13} = \frac{R_{+6} + R_{-6} + 2\sum_{i=-5}^{+5} R_i}{24} , \qquad (2)$$

where R_{+6} is the monthly mean sunspot number 6 months ahead of the month of interest, R_{-6} is the monthly mean sunspot number 6 months behind the month of interest, and $\sum_{i=-5}^{+5} R_i$ the sum of the monthly mean sunspot numbers 5 months either side and including the month of interest. Thus, \overline{R}_{13} values are always running about 6 months behind the R_Z numbers and about 7 months or so behind calendar time. \overline{R}_{13} can be based on either R_Z (now R_I) or R_A values, although usually it is the former.

A smoothed number can also be determined for F_{2800} on the basis of equation (2); the result is denoted here as \overline{F}_{13} . \overline{R}_{13} and \overline{F}_{13} are usually the more appropriate parameters to use, especially when performing statistical-type studies or estimating solar activity in future epochs, since their behavior is much smoother in appearance.

B. The Schematic Sunspot Cycle

Sunspot cycles, as plotted using smoothed sunspot numbers versus time, often manifest themselves as a curve rising from some minimum \overline{R}_{13} value to a higher maximum \overline{R}_{13} value and then declining to some other minimum \overline{R}_{13} value marking the end of the cycle. The minimum \overline{R}_{13} value at the start of a cycle is denoted here as \overline{R}_{MIN} or \overline{R}_{MIN} (SCN), where SCN means "sunspot cycle number." Similarly, the \overline{R}_{13} value at cycle maximum is denoted \overline{R}_{MAX} or \overline{R}_{MAX} (SCN) and the \overline{R}_{13} value at the end of the cycle is denoted \overline{R}_{MIN} (SCN + 1).

Sunspot cycles begin and end at minimum \overline{R}_{13} occurrence, by international convention. The reasons for the convention are primarily that solar activity, for example number of solar flares versus time, is higher near solar maximum than at solar minimum, indicating a cyclic nature; that sunspots, usually occurring in groups as "leaders" and "followers" (owing to solar rotation), undergo a hemispheric magnetic polarity reversal near solar minimum (e.g., northern hemispheric sunspots having leading spots of predominantly positive magnetic field during a cycle will have leading spots of predominantly negative magnetic field during the subsequent cycle); and that "new cycle" sunspots tend to be at higher average solar latitude at solar minimum and at progressively lower average latitude as the cycle progresses revealing the phenomenon known as "latitude drift," discovered by Christopher Carrington about 1858 and frequently referred to as Spörer's law, in honor of Gustav Spörer who also investigated the phenomenon. (Latitude drift is often illustrated by use of Maunder "butterfly diagrams," so called because their appearance bears a resemblance to the shape of butterfly wings.)

The time of \overline{R}_{MIN} occurrence is denoted here as $t(\overline{R}_{MIN})$ and the time of \overline{R}_{MAX} occurrence as $t(\overline{R}_{MAX})$. The difference between $t(\overline{R}_{MAX})$ and $t(\overline{R}_{MIN})$ for a cycle is called the ascent period and is denoted ASC; the difference between $t(\overline{R}_{MAX})$ and subsequent cycle $t(\overline{R}_{MIN})$ is the descent period, denoted DES. During ASC, sunspot number is increasing and, during DES, sunspot number is decreasing. The sum of ASC and DES for a cycle is the cycle duration or the minimum-to-minimum period, denoted MIN-MIN PERIOD. Similarly, the time from $\overline{R}_{MAX}(SCN)$ occurrence to $\overline{R}_{MAX}(SCN+1)$ occurrence is called the maximum-to-maximum period and is denoted MAX-MAX PERIOD. The time from \overline{R}_{MIN} occurrence to the \overline{R}_{13} value midway between \overline{R}_{MIN} and \overline{R}_{MAX} is denoted ASC_{CHM} (where the subscript CHM means "corrected half maximum"); the \overline{R}_{13} value at this point is denoted \overline{R}_{CHM} . \overline{R}_{CHM} is defined as

$$\overline{R}_{CHM} = \frac{\overline{R}_{MIN} + \overline{R}_{MAX}}{2} , \qquad (3)$$

where the terms \overline{R}_{MIN} and \overline{R}_{MAX} have their usual meanings. The time interval that a cycle is at or above \overline{R}_{CHM} is denoted D_{CHM} .

While $t(\overline{R}_{MAX})$ is usually reckoned in time space, relative to \overline{R}_{MIN} occurrence, it should be noted that it can also be reckoned in phase space (based on the MIN-MIN PERIOD). That is, denoting the phase of the cycle when \overline{R}_{MAX} occurs as Φ_{MAX} , the following expression is obtained:

$$\Phi_{\text{MAX}} = \frac{\text{ASC}}{\text{MIN-MIN PERIOD}} \qquad (4)$$

Similarly, time of \overline{R}_{MIN} (SCN + 1) occurrence in phase space (based on MAX-MAX PERIOD) can be reckoned. Denoting it as Φ_{MIN} , the following expression is obtained:

$$\Phi_{\text{MIN}} = \frac{\text{DES}}{\text{MAX-MAX PERIOD}} \tag{5}$$

Figure 1 schematically illustrates all aforementioned parameters.

Several other parameters of importance are reckoned using the derivative of the curve shown in Figure 1. These parameters are schematically illustrated in Figure 2. The time difference between \overline{R}_{MIN} occurrence and the value of greatest positive change in \overline{R}_{13} value is denoted t_{GPV} , where GPV means

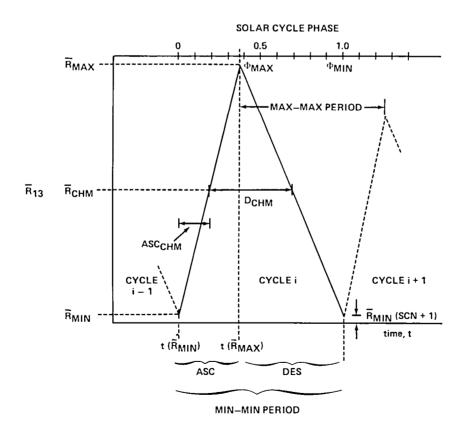


Figure 1. Schematic sunspot cycle showing selected parameters based on smoothed sunspot number versus time.

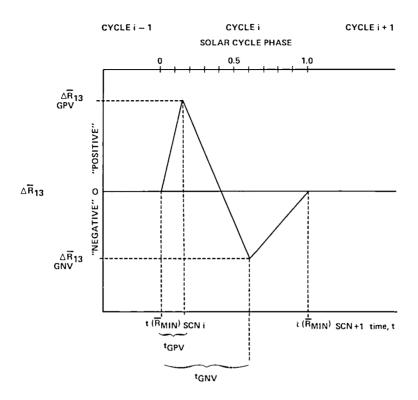


Figure 2. Schematic sunspot cycle showing selected parameters based on rate of change of smoothed sunspot number versus time.

"greatest positive value." The value of this change is denoted ${}_{GPV}^{\Delta}$ \overline{R}_{13} and the \overline{R}_{13} value at t_{GPV} is denoted \overline{R}_{13} (t_{GPV}). Similarly, the time difference between \overline{R}_{MIN} occurrence and the value of greatest negative change in \overline{R}_{13} value is denoted t_{GNV}^{Δ} , where GNV means "greatest negative value." The value of this change is denoted t_{GNV}^{Δ} \overline{R}_{13} and the \overline{R}_{13} value at t_{GNV}^{Δ} is denoted \overline{R}_{13} (t_{GNV}^{Δ}).

Another set of parameters, based upon areas under the R_Z versus t curve, may be of use. These include $\sum_{t=0}^{12} R_Z(t)$, $\sum_{t=0}^{18} R_Z(t)$, and $\sum_{t=0}^{24} R_Z(t)$. Here, R_Z is summed from \overline{R}_{MIN} occurrence to some point

in time; for example, $\sum_{t=0}^{12} R_Z(t)$ represents the sum of R_Z from t = 0 (R_Z value at \overline{R}_{MIN} occurrence) to

t=12 or 12 months into the cycle. These parameters will be compared in a later section to determine their "predictive" capability; that is, using an areal portion of a cycle (or one of the other parameters; e.g., \overline{R}_{MIN}) to estimate later occurring cycle parameters (e.g., \overline{R}_{MAX}). While sunspot cycles may not always fit this schematic picture, that is a simple rise to maximum followed by a simple fall to subsequent cycle minimum, even more complicated cycles — e.g., cycles marked by "bumps" in the rise and/or fall portion — can be crudely mapped by use of the aforementioned parameters.

Before going any further with a discussion of these cycle parameters, a word should be given about "sunspot cycle number" (dubbed SCN). The present cycle (SCN 21) began in June 1976 and

peaked in December 1979 (based on \overline{R}_{13} values). It is now decreasing towards the next cycle minimum, to occur in the mid-to-late 1980's. The Solar Maximum Mission is a satellite, launched in February 1980, which investigated the near maximum phase of this solar cycle. In contrast, Skylab flew during the declining portion of cycle 20. Virtually all orbital space solar physics has occurred over the last 20 years, corresponding to only about one full solar cycle and part of another. SCN 8 corresponds to the cycle reported by Schwabe in 1843. It began in November 1833. SCN 3 corresponds to the time period of the American Revolution, and SCN 0 to ca. 1750. Sunspot cycle numbers have been documented back to ca. 1705 corresponding to SCN-4 (see Allen [18]), but recall that the quality of the data is suspect prior to SCN 8. This report is based strictly on SCN's 8 through 20 corresponding to the period November 1833 to June 1976. Based on this data set, estimates for the aforementioned parameters will be made for cycle 21 and compared to observed values when those values are known.

III. RESULTS

Under this heading, the presentation of the material, for convenience, is subdivided into six subsections. Note that these subsections, as well as much of the remainder of the report, are primarily statistical in nature; hence, many equations of similar form are presented. These linear equations take the form y = a + bx. The reader is reminded that this comparison is not done to imply a real physical relationship between the parameters x and y, although one cannot be precluded, but instead to show their numerical behavior, one relative to the other, for a specific data set. How good an estimate obtained for y relative to x is dependent upon the magnitude of the correlation coefficient (r) and the standard error of estimate (S_{yx}) . When r is low, the mean (x) and standard deviation (s) will suffice; however, when r is high (e.g., $|r| \ge 0.5$), estimates based on the linear equations are the more appropriate to use. (Both s and S_{yx} are measures of the statistical spread of the data; when r is large, S_{yx} is smaller than s, yielding a better estimate for a parameter.) Two additional reminders should be emphasized: First, the reader should be cautioned that the linear equations are based on specific data sets covering finite periods of time, short on the cosmic time scale; and second, because of the rather large number of similar equations with many similar symbols the reader may encounter some difficulty in following the presentation of the results. In an effort to alleviate some of the potential chance of confusion and to serve as quick reference summaries, figures and tables have been amply provided.

A. Linear Regression Equations for Selected Measures of Solar Activity

As mentioned in Section II, there are several commonly used measures of solar activity. These include (but are not limited to) R_A , R_Z , \overline{R}_{13} , F_{2800} , and \overline{F}_{13} , all defined as before. Since the most convenient parameter is probably the provisional value of R_Z , it may be of interest to compare the other parameters with it.

Following basic statistical methods (e.g., Downie and Heath [19]) and using the data set spanning April 1954 through June 1982 (SCNs 19 and 20 and part of cycle 21), a linear-regression fit between values of R_A and the provisional values of R_Z is obtained. This fit is depicted in Figure 3. The Pearson correlation coefficient r is calculated to be 0.991 and the standard error of estimate S_{yx} is equal to 7.30. Thus, 50 percent of the data points are within 4.9 units of the regression line (i.e., 0.675 S_{yx}) and 90 percent are within 12 units (i.e., 1.645 S_{yx}); the 99 percent confidence limits (i.e., 2.575 S_{yx}) are approximately 18.8 units about the regression line. The regression equation is

$$R_A = -0.012 + 0.944 R_Z \text{ (PROVISIONAL)}$$
, (6)

where the two numbers on the right side of equation (6) are the regression coefficients a and b, when the equation is of the form y = a + bx; i.e., a = -0.012 and b = 0.944.

* DENOTED R_I BEGINNING IN DECEMBER 1980; DERIVED BY SUNSPOT INDEX DATA CENTER, BRUSSELS, BELGIUM

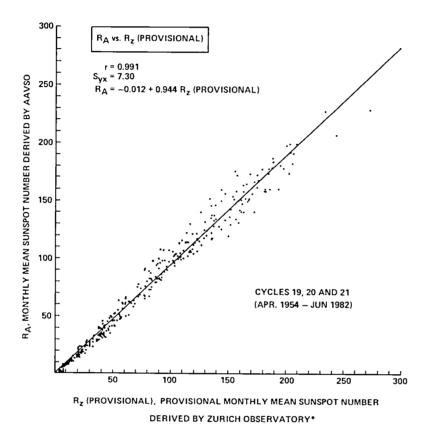


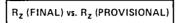
Figure 3. Monthly mean sunspot number derived by AAVSO versus provisional monthly mean sunspot number derived by Zurich Observatory.

A regression fit between final values and provisional values of R_Z is shown in Figure 4. It is based on the April 1954 through December 1981 timeframe. One observes r = 0.999, $S_{VX} = 2.65$, and

$$R_Z$$
 (FINAL) = 1 + 0.999 R_Z (PROVISIONAL) . (7)

Equation (7) clearly shows that the difference between provisional and final values of R_Z is very small; using 99 percent confidence limits and a provisional value of R_Z equal to 100 yields a spread of about ± 6.8 percent.

*DENOTED R_I BEGINNING IN DECEMBER 1980; DERIVED BY SUNSPOT INDEX DATA CENTER, BRUSSELS, BELGIUM



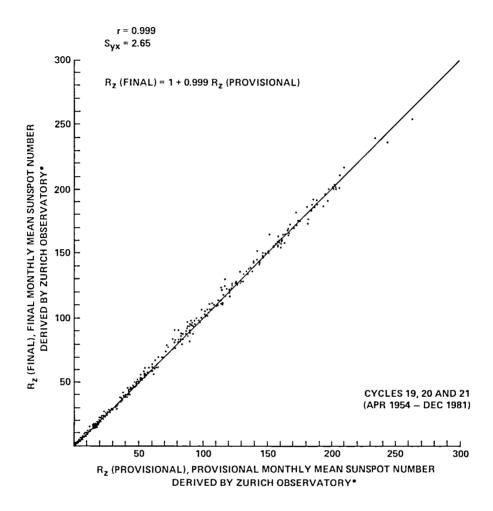


Figure 4. Final monthly mean sunspot number versus provisional monthly mean sunspot number.

A regression fit between 2800-MHz radio emission data, adjusted to 1 AU and corrected for bursts (recall F_{2800}), and provisional values for R_Z for the time period April 1954 through June 1982 results in Figure 5. One observes r = 0.965, $S_{VX} = 14.11$, and

$$F_{2800}$$
 (ADJUSTED TO 1 AU) = 62.986 + 0.879 R_Z (PROVISIONAL) . (8)

Combining equations (7) and (8) and solving for R_Z (FINAL) results in

$$R_Z$$
 (FINAL) = -70.66 + 1.14 F_{2800} (ADJUSTED TO 1 AU) . (9)

This equation is equivalent in form to that given in <u>Solar-Geophysical Data</u>—<u>Explanation of Data Reports</u> (e.g., February 1982); that equation is given as

$$R_S = -62 + 1.08 S_A$$
 , (10)

where S_A is the same as F_{2800} and R_S is the resultant sunspot number using F_{2800} as the variable. (R_S is equivalent to $R_Z(FINAL)$.) It should be pointed out that equation (10) is based on the much larger data set 1947 through 1979. (F_{2800} has been routinely measured since 1947.)

*DENOTED R₁ BEGINNING IN DECEMBER 1980; DERIVED BY SUNSPOT INDEX DATA CENTER, BRUSSELS BELGIUM

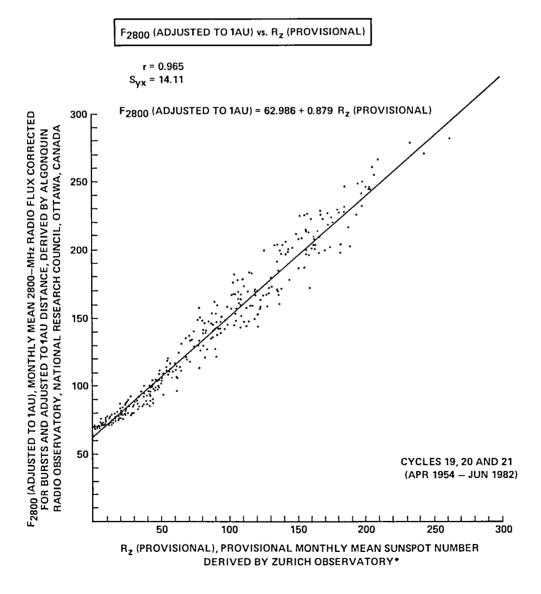


Figure 5. Monthly mean 2800-MHz radio flux corrected for bursts and adjusted to 1 AU versus provisional monthly mean sunspot number.

The large standard error of estimate identified for equation (8) (S_{yx} = 14.11) results because, while both parameters are measures of solar activity and show very similar trends (i.e., a "bump" in one curve has a corresponding "bump" in the other parameter curve; Wilson [13]), the actual "peaking" of the two curves may be dissimilar; thus, a large scatter is introduced. As an example, based on a comparison of \overline{R}_{13} and \overline{F}_{13} , it is found that cycle 19 had peaks in both curves at the same point in time; however, both cycles 20 and 21 reveal \overline{R}_{13} to peak prior to \overline{F}_{13} by about 1.5 years (20 and 17 months, respectively). Figure 6 depicts cycles 19, 20 and a portion of cycle 21, in terms of \overline{F}_{13} and \overline{R}_{13} as a function of time. In addition to comments already made, one should note that \overline{F}_{MIN} and \overline{R}_{MIN} always seem to coincide in time, and that there has been an apparent flattening of \overline{F}_{13} near peak in the two most recently observed cycles, which is in sharp contrast to its behavior in cycle 19.

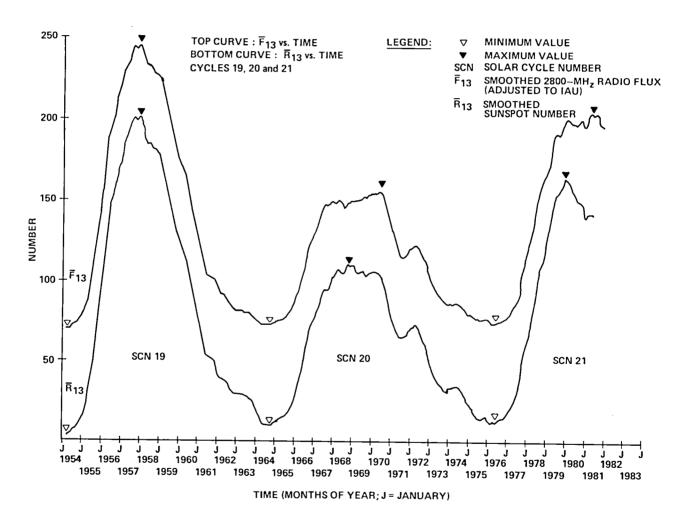


Figure 6. Smoothed sunspot number versus time for cycles 19, 20, and 21.

Figure 7 illustrates a comparison of smoothed sunspot number (\overline{R}_{13}) and "final" R_Z values. The regression fit is based on the November 1833 through June 1976 data set. One observes r = 0.949, $S_{yx} = 12.84$, and

$$\overline{R}_{13} = 6.40 + 0.88 R_Z \text{ (FINAL)}$$
 (11)

Equation (11) can be combined with equations (7) and (9), resulting in

$$\overline{R}_{13} = 7.28 + 0.88 R_Z \text{ (PROVISIONAL)}$$
 (12)

and

$$\overline{R}_{13} = -55.78 + F_{2800} \text{ (ADJUSTED TO 1 AU)} ,$$
 (13)

respectively.

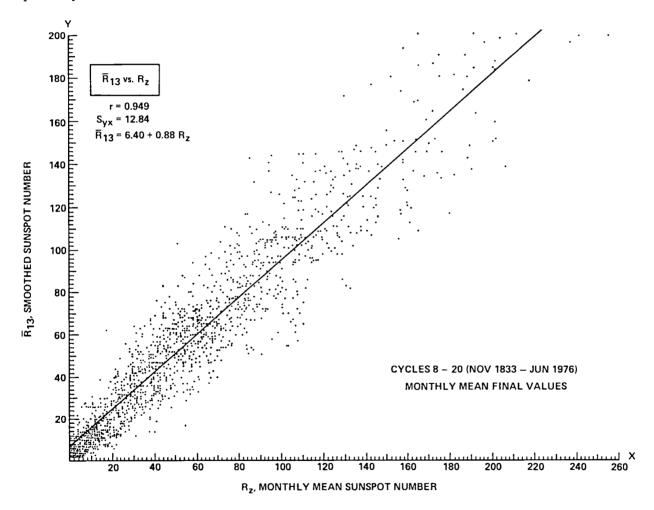


Figure 7. Smoothed sunspot number versus final monthly mean sunspot number.

A comparison of \overline{F}_{13} with F_{2800} is shown in Figure 8. The regression fit is based on the April 1954 through December 1981 data set. One observes r = 0.973, $S_{VX} = 12.04$, and

$$\overline{F}_{13} = 9.208 + 0.930 F_{2800} \text{ (ADJUSTED TO 1 AU)}$$
 (14)

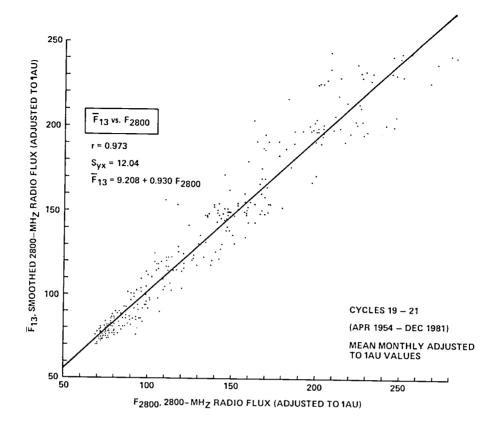


Figure 8. Smoothed 2800-MHz radio flux corrected for bursts and adjusted to 1 AU versus monthly mean 2800-MHz radio flux corrected for bursts and adjusted to 1 AU.

Combining equations (13) and (14) results in

$$\overline{F}_{13} = 61.083 + 0.930 \ \overline{R}_{13}$$
 (15a)

and

$$\overline{R}_{13} = -65.681 + 1.075 \overline{F}_{13}$$
 (15b)

Equation (15a) can be combined with equation (12), resulting in

$$\overline{F}_{13} = 67.853 + 0.818 R_Z \text{ (PROVISIONAL)}$$
, (16)

or equation (15a) can be combined with equation (8), resulting in

$$\overline{F}_{13} = 67.785 + 0.817 R_Z \text{ (PROVISIONAL)}$$
 (17)

(The difference in coefficients for equations (16) and (17) is due to roundoff effects.) Table 1 summarizes the regression coefficients and standard error of estimates using R_Z (PROVISIONAL) as the variable.

TABLE 1. LINEAR REGRESSION COEFFICIENTS FOR SELECTED PARAMETERS
BASED ON COMPARISON TO PROVISIONAL VALUE OF
MONTHLY MEAN SUNSPOT NUMBER

Y VARIABLE*	a	b	s_{yx}	EQ. NO.
R _z (FINAL)	1	0.999	2.65	7
R 13	7.28	0.88	15.17	12
F ₂₈₀₀ (ADJUSTED TO 1 AU)	62.986	0.879	14.11	8
F ₁₃	67.785	0.817	13.12	17

^{*}X VARIABLE IS R_z (PROVISIONAL)

B. Parametric Values Versus SCN

Appendix A contains sunspot number information for cycles 8 through 21 in terms of R_Z (FINAL) and \overline{R}_{13} based on the parameters t (elapsed time since \overline{R}_{MIN} occurrence) and T (elapsed time since \overline{R}_{MAX} occurrence). Also given are values of phase (Φ), both in terms of t and T, and $\Delta \overline{R}_{13}$. At the right is a "Notes" column which identifies the SCN and such parameters as \overline{R}_{MIN} , \overline{R}_{MAX} , \overline{R}_{13} and \overline{R}_{13} occurrence. At the left is a "Date" column which gives the month and year for \overline{R}_{MIN} and \overline{R}_{MAX} occurrence within a cycle. Appropriate information can be extracted from Appendix A to form Table 2.

In Table 2, SCN is identified in the leftmost column and sunspot minimum and maximum occurrence dates (in month and year of occurrence) are given adjacent to it. The remaining parameters are identified in columns to the right of the occurrence dates. Values for SCN 21 are also given but have not been used in computation of mean values and standard deviations, which are given for each parameter in their proper columns. Thus, the mean \overline{R}_{MIN} for cycles 8 through 20 is about 5.2 with a standard deviation of 2.7; the mean \overline{R}_{MAX} is about 116.2 with a standard deviation of 36.7; etc. The range is identified, in terms of extreme values (HIGH and LOW) for each parameter, at the bottom of Table 2. Thus, \overline{R}_{MIN} values range from 1.5 to 10.5, \overline{R}_{MAX} values from 64.2 to 201.3, etc. It is noted that SCN 9 had an extremely large ASC_{CHM} (= 44); so, mean values and standard deviations are given for ASC_{CHM} both including and excluding the ASC_{CHM} value for SCN 9. Therefore, the mean ASC_{CHM} including SCN 9 is about 26.3 with a standard deviation of 5.8, and the mean ASC_{CHM} excluding SCN 9 is about 24.8 with a standard deviation of 2.9.

TABLE 2. SUMMARY OF SELECTED SUNSPOT CYCLE PARAMETERS FOR SCN 8 THROUGH 21

SOLAR CYCLE NUMBER (SCN)	SUNSPOT MINIMUM OCCURRENCI	SUNSPOT MAXIMUM E OCCURRENCE	R _{MIN}	R _{MAX}	Ř _{CHM}	ASC _{CHM}	D _{CHM}	ASC	DES	MIN-MIN PERIOD	MAX-MAX PERIOD	Фмах	Фмін	¹ GPV	∆ GPV Ř 13	R ₁₃ (^t GPV)		∆ gnv ^R 13	R ₁₃ (t _{GNV}) Ř _{MIN} (SCN+1)
	NOV 1833	MAR 1837	7,3	146,9	77.1	23	53	40	76	116	131	0,345	0,580	31	+9.5	116,1	59	-6,2	93.6	10.5
9	JUL 1843	FEB 1848	10,5	132,0	71.3	44	39.	55	54	149	144	0,369	0,653	45	+8,4	83,1	71	-5,9	99.0	3.2
10	DEC 1855	FEB 1860	3,2	97,9	50.6	29	57	50	85	135	126	0,370	0,675	34	+4,1	67,6	69	-4,2	73.7	5.2
11	MAR 1867	AUG 1870	5,2	140,5	72,9	28	46	41	100	141	160	0,291	0,625	30	+9,4	84.4	72	-5,2	98,3	2.2
12	DEC 1878	DEC 1883	2.2	74,6	38,4	22	63	60	74	134	121	0,448	0,612	14	+4.1	19,8	83	-4,1	45.0	5.0
13	FEB 1890	JAN 1894	5,0	87,9	46,5	20	54	47	96	143	145	0,329	0,662	17	+4.6	37,9	58	-3.6	71,3	2.7
14	JAN 1902	FEB 1906	2,7	64,2	33,5	23	72	49	89	138	138	0,355	0,645	39	+3,9	56,6	85	-4,8	46,4	1.5
15	JULY 1913	AUG 1917	1.5	105,4	53,5	27	50	49	71	120	128	0,408	Q.555	43	+8,5	81,2	76	-5,3	56,7	5.6
16	JULY 1923	APR 1928	5,6	78,1	41.9	24	59	57	65	122	108	0,467	0,602	23	+6,2	40,9	83	-5,6	39,2	3.5
17	SEP 1933	APR 1937	3.5	119,2	61,4	29	57	43	82	125	121	0,344	0,678	39	+6.4	101,2	73	-4.7	84,3	7.7
18	FEB 1944	MAY 1947	7,7	151,8	79,8	27	51	39	83	122	130	0,320	0,638	33	+8.6	117,6	73	-6,9	106,4	3.4
19	APR 1954	MAY 1958	3,4	201,3	102,4	23	54	47	79	126	128	0.373	0,617	22	+10.8	98.5	75	-6,2	108,6	9.6
20	OCT 1964	NOV 1968	9,6	110,6	60,1	23	74	49	91	140	133	0,350	0,684	21	+6,4	50,3	73	-5,3	89,4	12.2
21	JUN 1976	DEC 1979	12,2	164,5	88.4	24	-	42	-	-	-	-	-	24	+8.1	89.3	-	-	-	-
MEAN ICYCLES	3–20):		5,18	116,18	60,72	26,31 (24,83)*	56,08	48.1	5 83,4	6 131,62	131,77	0,367	0,633	30,08	+6,99	73,48	73,08	-5,23	77,84	5.56
STANDARD DEV	TATION (CYCLE	ES 8-20)	2,74	36,66	18.77	5,81 (2,88)	9,23	6,2	1 9,9	9,95	12,48	0,048	0,038	9,61	2.27	29,74	7.84	0,91	23,40	3.28
EXTREME VALU	ES (CYCLES 8-	20): HIGH	10,5	201,3	102,4	44	74	60	100	149	160	0,467	0.684	45	+10,8	117.6	85	-3,6	108,6	12.2
		LOW	1,5	64,2	33.5	20	39	39	65	116	106	0,291	0,555	14	+3,9	19,8	58	-6,9	39,2	1.5

^{*}EXCLUDES SCN 9

Table 3 serves as a "quick-reference" chart for determining which SCNs are above (+) or below (-) mean value in terms of the aforementioned sunspot cycle parameters. The mean value for a parameter is identified adjacent to its corresponding parameter. SCN 21 is again included for completeness. Since SCN 21 is only partially complete, a number of its parameters remain unknown; so, for these values, question marks (?) are given.

Figures 9 A, B, and C graphically illustrate the variation in parametric value with SCN in terms of ordinary number value. Figures 10 A, B, and C display the variation in terms of mean and standard deviation values (i.e., a residual RES, where RES equals the difference quantity SCN parametric value minus mean parametric value divided by the parametric standard deviation; the mean value is given for each parameter as \overline{x} and the standard deviation as s).

Figure 11 gives number of occurrences (histograms) versus parametric value in terms of \overline{x} and s (graphed in units of 0.5 s). Thus, it is a graphical illustration of the distribution for each parameter. Since only 13 cycles are included in this study, a "normal" distribution is not strongly suggested for most of the parameters shown. Interestingly, MAX-MAX PERIOD looks somewhat like a normal distribution while MIN-MIN PERIOD looks more like a "bi-modal" distribution suggesting, perhaps, that MIN-MIN PERIOD can be subdivided into two cycle groups: a short-period cycle group and a long-period cycle group. This latter suggestion is more dramatically illustrated in a plot of phase (Φ) versus elapsed time since \overline{R}_{MIN} occurrence (t), as is shown in Figure 12. If MIN-MIN PERIOD is regarded to be comprised of two distinct cycle groups, it is found that Φ and t appear to be related for these groups in the following manner:

LONG-PERIOD CYCLES:
$$\Phi = 0.00714 \text{ t}$$
 (18a)

and

SHORT-PERIOD CYCLES:
$$\Phi = 0.00821 \text{ t}$$
 (18b)

Figure 12 shows that \overline{R}_{MAX} always occurs (based on cycles 8 through 20) in the box bounded by t equal to 39 and 60 and Φ equal to 0.29 and 0.47. Cycle 21 reached peak in December 1979 yielding an ascent period of 42 months. Thus, if cycle 21 is a short-period cycle, the phase at \overline{R}_{MAX} would have been 0.34, suggesting a MIN-MIN PERIOD of about 122 months, give or take about 5 months (based on range). This suggests that \overline{R}_{MIN} for cycle 22 may occur between December 1985 and October 1986, with the best guess being about May 1986. On the other hand, if cycle 21 is a long-period cycle, the phase at \overline{R}_{MAX} would have been 0.30, suggesting a MIN-MIN PERIOD of about 140 months, give or take about 7 months. This suggests that \overline{R}_{MIN} for cycle 22 may occur between June 1987 and August 1988, with the best guess being about January 1988. Thus, two predictions for \overline{R}_{MIN} -occurrence date for cycle 22 can be established, but only one can be correct if cycles are distributed in the aforementioned manner. (Based on average period length, cycle 22 would be expected to begin about May 1987.) As will be seen in a later section, it seems more plausible that cycle 21 is a short-period cycle. Thus, cycle 22 may begin as early as 1986.

TABLE 3. QUICK-REFERENCE TABLE FOR DETERMINING ABOVE (+) AND BELOW (-) PARAMETRIC MEAN VALUE CYCLES

SOLAR CYCLE NUMBER (SCN)

PARAMETER	MEAN VALUE	8	9	10	11	12	13	14	15	16	17	18	19	20	21
\overline{R}_{MIN}	5.18	+	+	_	+	-	-	_	_	+	-	+	-	+	+
\overline{R}_{MAX}	116.18	+	+	-	+	-	_	_	_	-	+	+	+	_	+
R _{CHM}	60.72	+	+	_	+	_	-	_	_	_	+	+	+	-	+
ASC _{CHM}	26.31	_	+	+	+	_	_	_	+	-	+	+	-	-	_
D _{CHM}	56.08	_	-	+	-	+	-	+	_	+	+	-	-	+	?
ASC	48.15	_	+	+		+	-	+	+	+	-	-	-	+	-
DES	83.46	-	+	+	+	_	+	+	_	_	-	-	-	+	?
MIN-MIN PERIOD	131.62	_	+	+	+	+	+	+	-	_	_	-	-	+	?
MAX-MAX PERIOD	131.77	-	+	_	+		+	+	_	_	-	-	-	+	?
$^{\Phi}$ MAX	0.367	_	+	+		+	_	_	+	+	-	-	+	_	?
Φ MIN	0.633	_	+	+	_	_	+	+	_	-	+	+	-	+	?
^t GPV	30.08	+	+	+	-	_	-	+	+	-	+	+	-	-	_
GPVR13	+6.99	+	+	_	+	_	_	-	+	_	_	+	+	-	+
R ₁₃ (t _{GPV})	73.48	+	+	_	+	_	-	-	+	_	+	+	+	_	+
^t GNV	73.08	_	-	_	-	+	-	+	+	+	-	-	+	-	?
GNV ^R 13	-5.23	_		+	+	+	+	+	_	_	+	_	_	_	?
R ₁₃ (t _{GNV})	77.84	+	+	_	+	_	-	_	-	-	+	+	+	+	?
R _{MIN} (SCN + 1)	5.56	+	_	+	-	_	-	-	+	-	+	_	+	+	?

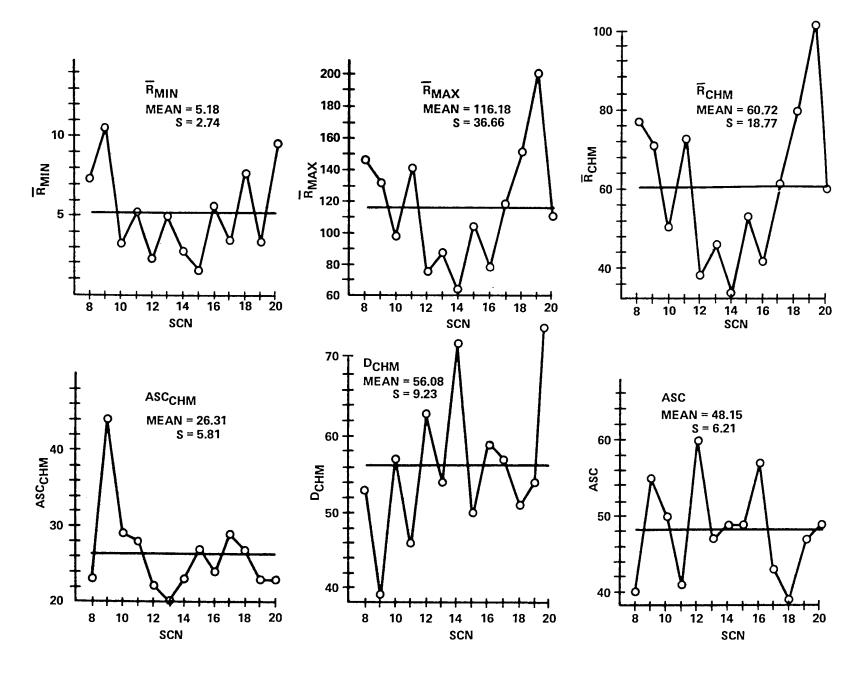


Figure 9. Parametric values versus SCN (in terms of ordinary number value).

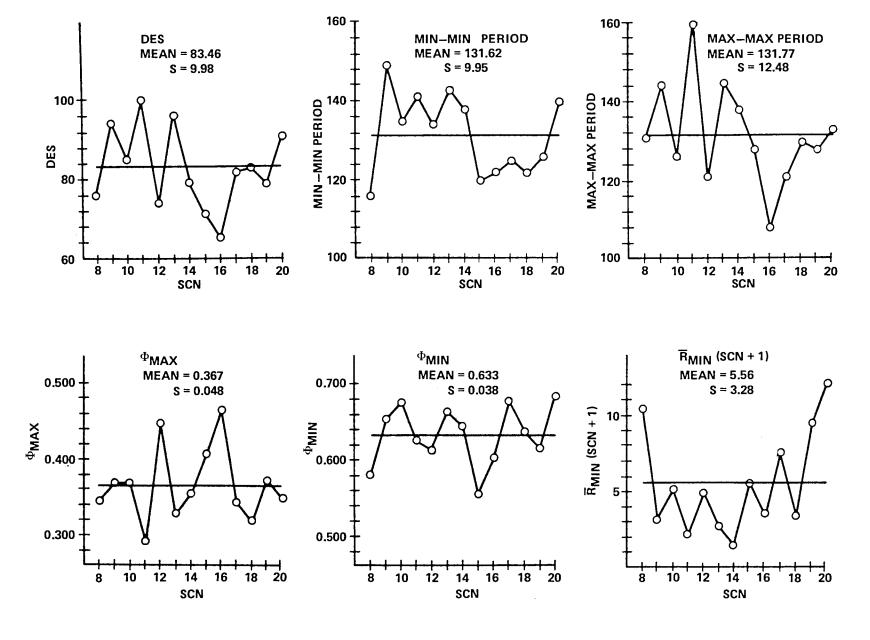


Figure 9. (Continued)

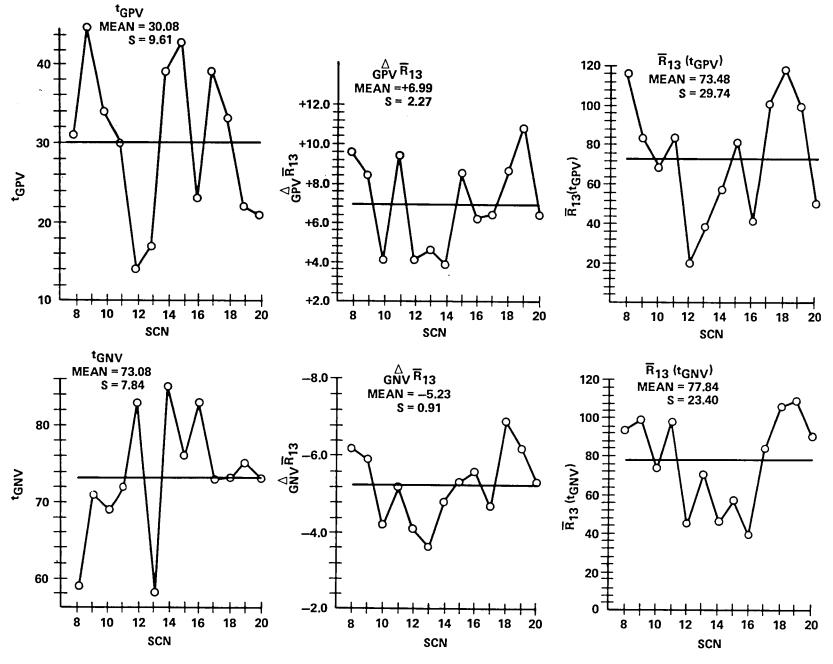


Figure 9. (Concluded)

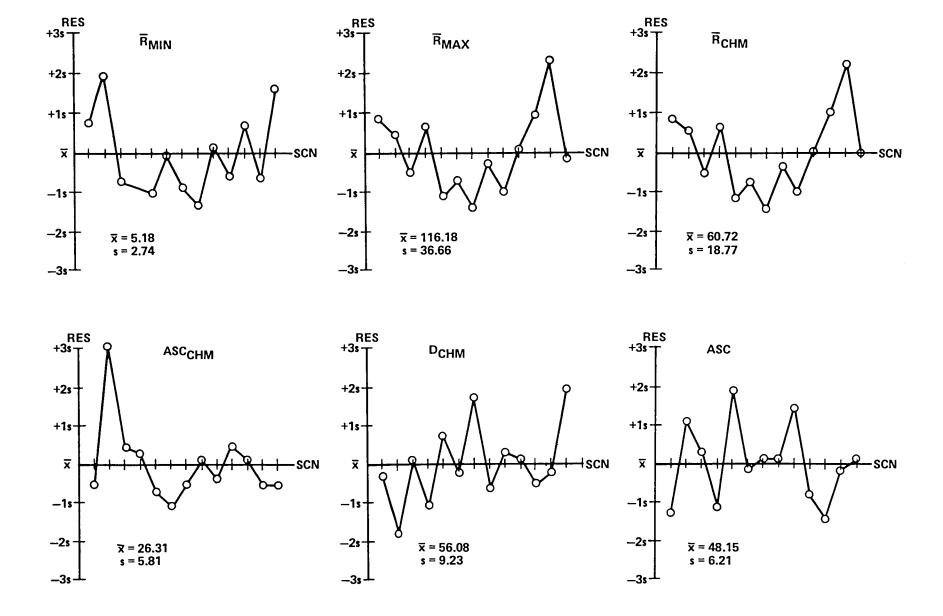


Figure 10. Parametric values versus SCN (in terms of mean and standard deviation value).

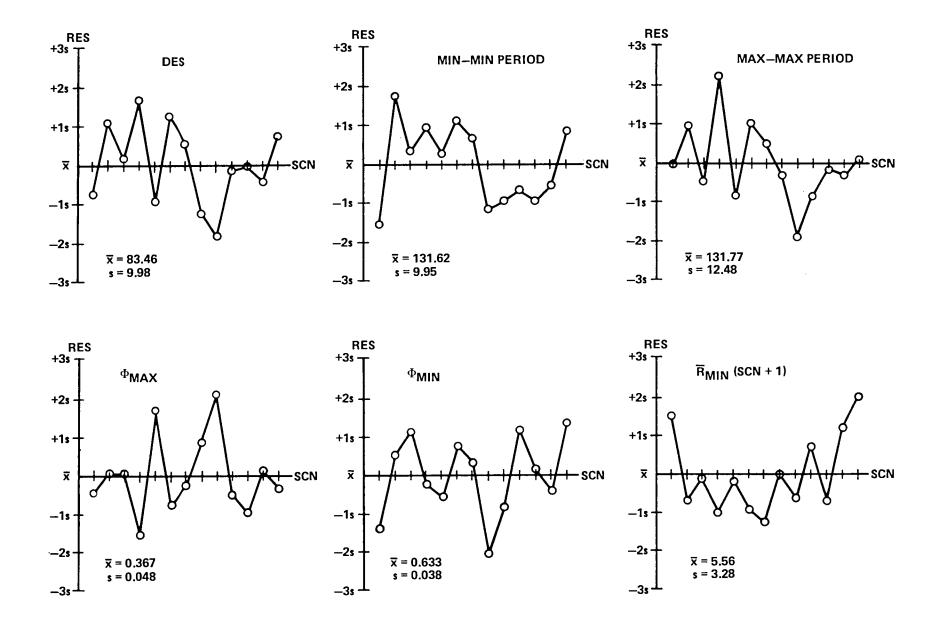


Figure 10. (Continued)

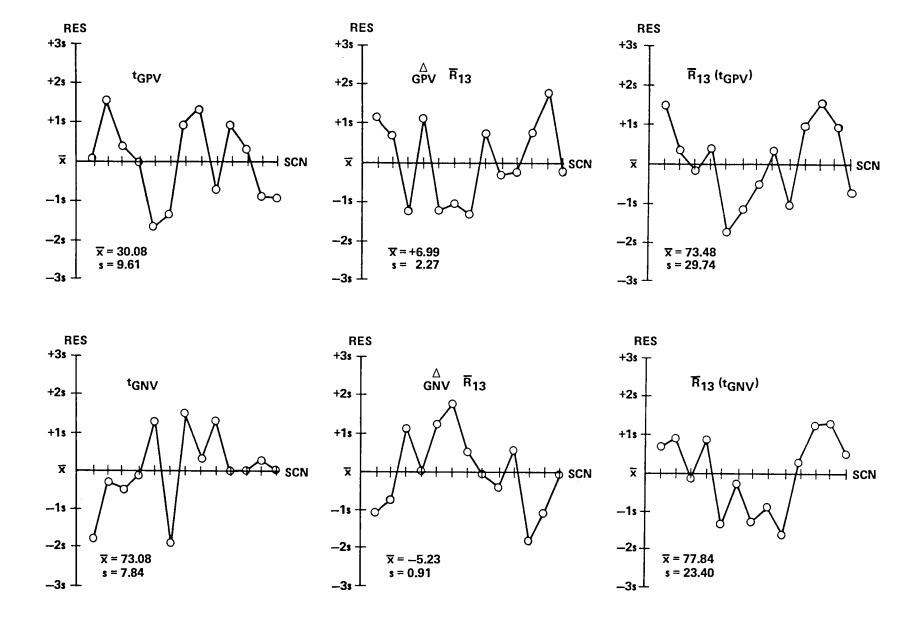


Figure 10. (Concluded)

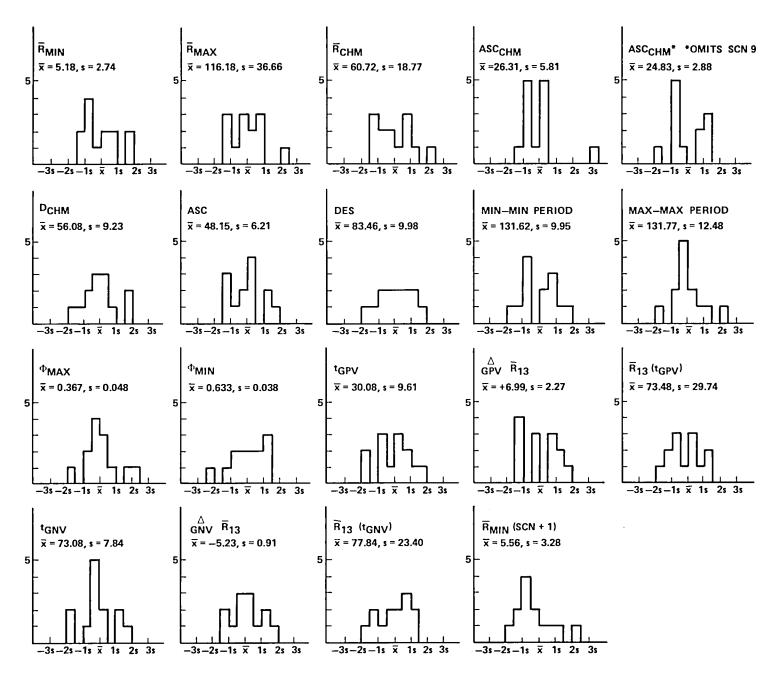
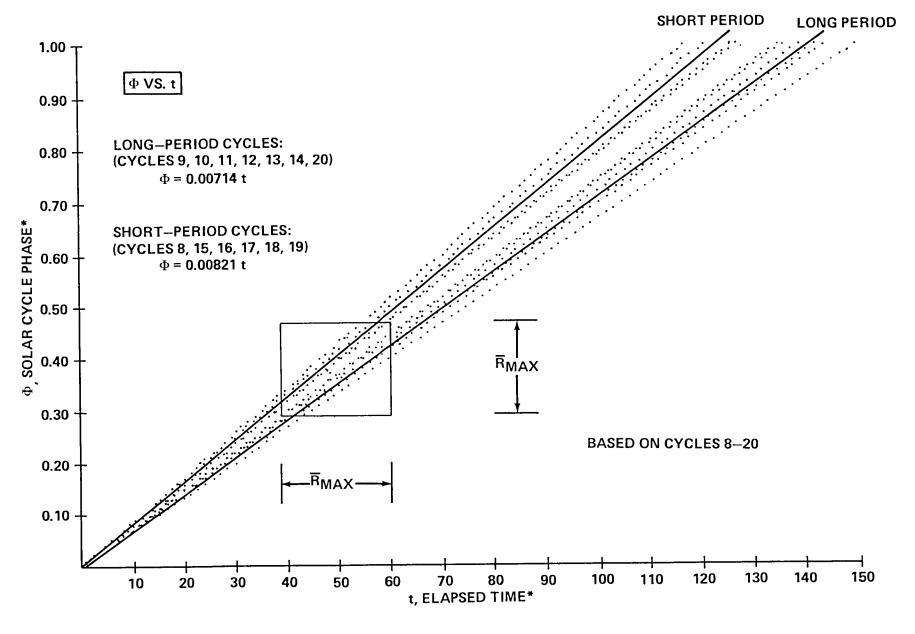


Figure 11. Histograms of parametric values (in units of 0.5 s).



*FROM RMIN OCCURRENCE

Figure 12. Sunspot cycle phase versus time (suggesting long- and short-period groupings).

C. Mean Parametric Values Based on Selected Groupings of SCN

Table 4 lists parametric mean values (\bar{x}) and standard deviations (s) for several selected groupings of SCN. The parameters are identified in the leftmost column and mean values and standard deviations, based on cycles 8 through 20, are shown adjacent to the parameters for comparison. Six series of columns (2 or 3 columns each) representing six selected subgroupings within cycles 8 through 20 appear to the right. The first series identifies the cycles (by SCN), \overline{x} , and s for the group "cycles with parametric values $\geq \overline{x}_{8-20}$." That is, as an example, cycles with \overline{R}_{MIN} greater than or equal to $\overline{x}_{8-20} = 5.2$ include SCNs 8, 9, 11, 16, 18, and 20; \overline{x} for this group is about 7.7 and s is about 1.9. The second series identifies the cycles, \overline{x} , and s for the group "cycles with parametric values $< \overline{x}_{8-20}$." Thus, continuing the example, cycles with \overline{R}_{MIN} less than about 5.2 include SCNs 10, 12-15, 17, and 19; \overline{x} for this group is about 3.1 and s is about 1.0. The third and fourth series gives \overline{x} and s values for the group of cycles with $\overline{R}_{MAX} \ge \overline{x}_{8-20} = 116.2$ (here, called HIGH- \overline{R}_{MAX} cycles) and those $< \overline{x}_{8-20} = 116.2$ (here, called LOW- \overline{R}_{MAX} cycles), respectively. The cycles included in these groupings are identified above \overline{x} and s. The last two series gives \overline{x} and s values for the group of cycles with MIN-MIN PERIOD $\geq \overline{x}_{8-20} = 131.6$ (here, called LONG-PERIOD cycles) and those $< \overline{x}_{8-20} = 131.6$ (here, called SHORT-PERIOD cycles), respectively. As with the previous series of groupings, the cycles comprising these groups are identified above their corresponding \overline{x} and s columns.

D. Mean R_Z and \overline{R}_{13} Curves Versus t

In the previous sections the parameters were determined individually within each solar cycle and then averaged to deduce mean values, standard deviations, and ranges, following the "schematic cycle" approach. A second approach, that of summing together $R_Z(t)_{SCN}$ or $\overline{R}_{13}(t)_{SCN}$ values for cycles 8 through 20 or selected groupings of cycles and computing means and standard deviations of R_Z and \overline{R}_{13} values as a function of t, can also be used to deduce parametric values. (Here, sum of $R_Z(t)_{SCN}$ means to sum all R_Z values as a function of t for the collection of SCNs of interest, whether cycles 8 through 20 or some other selected groupings of cycles. Similarly, sum of $\overline{R}_{13}(t)_{SCN}$ means to sum \overline{R}_{13} values as a function of t for the collection of SCNs of interest.) This second approach, an "epoch analysis" scheme, perhaps, offers a more eye-pleasing view of the cycle than the schematic view, in that it yields a continuous curve as a function of t over the cycle. (Remember, the schematic cycle only allows one to get estimates of \overline{R}_{13} values at selected times within a cycle or estimates of time/parameter value for selected parameters within a cycle.)

Figure 13 shows the mean $R_Z(t)$ curve which results when $R_Z(t)$ is summed for cycles 8 through 20 as a function of t (elapsed time since \overline{R}_{MIN} occurrence) and each $R_Z(t)$ sum is divided by 13 (the number of cycles included in this study). One observes $R_Z(MIN)$ at t=0 and $R_Z(MAX)$ at t=50. The values of $R_Z(t)$ at t=0 and t=50 are 4.1 and 113.5, respectively, and these minimum and maximum $R_Z(t)$ values are identified with light and dark triangles, respectively. Figure 14 plots the standard deviation of $R_Z(t)$ values as a function of t; i.e., $S_Z(t)$. It is seen that $S_Z(t)=0$ is about 3.0 and $S_Z(t)=0$ is about 44.5; the maximum $S_Z(t)=0$ value occurs at t=42 and has a value of about 55.3. The triangles correspond to minimum and maximum $S_Z(t)=0$ occurrences, as before. Since $S_Z(t)=0$ is one of the most easily

TABLE 4. PARAMETRIC VALUES FOR SELECTED GROUPINGS OF CYCLES

	CYCLE	S 8-20	CYCLES WI VALUES ≫		ETRIC	CYCLES WITH VALUES $< \overline{X}_2$		ETRIC	HIGH- CYC	R _{MAX}	LOW-	R _{MAX}	LO: PERIOD	NG- Cycles		ORT- CYCLES
PARAMETER	<u>x</u>	_	CONS	<u>X</u>	_	Stron	<u>x</u>		S: 8,9,11,		-	16,20		9-14,20		8,15-19
		<u>s</u>	SCNS	Δ.	<u>s</u>	SNCS	Δ	<u>s</u>	<u>x</u>	<u>\$</u>	<u>x</u>	<u>\$</u>	<u>X</u>	<u>s</u>	<u>x</u>	<u> </u>
RMIN	5.18	2.74	8,9,11,16,18,20	7.65	1.93	10,12-15,17,19	3.07	1.03	6.27	2.52	4.26	2.57	5.49	3.07	4.83	2.23
RMAX	116.18	36.66	8,9,11,17 — 19	148.62	25.81	10,12-16,20	88.39	15.83	148.62	25.81	88.39	15.83	101.10	26.31	133.78	39,10
RCHM	60.72	18.77	8,9,11,17 – 19	77.48	12.55	10,12-16,20	46.36	8.38	77.48	12.55	46.36	8.51	53.33	14.25	69.35	19.70
ASC _{CHM}	26.31	5.81	9 -11,15,17,18	30.67	6.02	8,12-14,16,19,20	22.57	1.18	29.00	7.09	24.00	2.83	27.00	7.56	25.50	2.29
DCHM	56.08	9.23	10,12,14,16,17,20	63,67	6.92	8,9,11,13,15,18,19	49.57	5.04	50.00	5.94	61.29	8.31	57.86	11.95	54.00	3.16
ASC	48.15	6.21	9,10,12,14-16,20	52.71	4.23	8,11,13,17-19	42.83	3,18	44.17	5.49	51.57	4.53	50.14	5.57	45.83	6.12
DES	83.46	9.98	9-11,13,14,20	92.50	4.86	8,12,15-19	75.71	5.90	85.67	8.50	81.57	10.74	89.86	7.88	76.00	6.32
MIN-MIN	131.62	9.95	9-14,20	140.00	4,72	8,15-19	121.83	3.29	129.83	11,42	133,14	8.18	140.00	4.72	121.83	3.29
MAX-MAX	131.77	12.48	9,11,13,14,20	144.00	9.10	8,10,12,15-19	124.13	7.03	135.67	12.84	128.43	11.12	138.14	12.11	124.33	7.97
Φ_{MAX}	0.367	0.048	9,10,12,15,16,19	0,406	0.039	8,11,13,14,17,18,20	0.333		0.340	0.028	0.390	0.049	0.359	0.044	0.376	
Φ_{MIN}	0.633	0.038 9	9,10,13,14,17,18,20	0.662	0.016	8,11,12,15,16,19	0.599	0.024	0.632	0.030	0.634	0.043	0.651	0.024	0.612	
^t GPV	30.08	9.61	8-10,14,15,17,18	37.71	4.86	11-13,16,19,20	21.17	5,01	33.33	7.23	27.29	10.48	28.57	10.78	31.83	7.67
^t GPV GPV ^R 13	+6.99	2.27	8,9,11,15,18,19	+9.20	0.83	10,12-14,16,17,20	+5.10	1.09	+8.85	1.34	+5,40	1.58	+5.84	2.10	+8.33	1.62
R ₁₃ (t _{GPV})	73.48	29.74	8,9,11,15,17-19	97.44	14,19	10,12-14,16,20	45.52	15.13	100.15	13.55	50.61	18.77		21.84	92.58	26.11
^t GNV	73.08	7.84	12,14-16,19	80.40	4.08	8-11,13,17,18,20	68.50	5.92	70.50	5.28	75.29	8.92	73.00	8.37	73.17	7.17
^t GNV GNVR13	-5.23	0.91	8,9,15,16,18-20	-5.91	0.53	10-14,17	-4.43	0.52	-5.85	0.72	-4.70	0.69	-4.73	0.74	-5.82	0.71
R ₁₃ (t _{GNV})	77.84	23.40	8,9,11,17-20	97.09	8.10	10,12-16	55.38	13,17	98.37	8.06	60.24	17.04		20.90	-3.62 81.47	25.54
R _{MIN} (SCN+1)	5.56	3.28	8,15,17,19,20	9.12	2.28	9-14,16,18	3.34	1.19	6.10	3.29	5.10	3.21	4.57	3.36	6.72	25.54

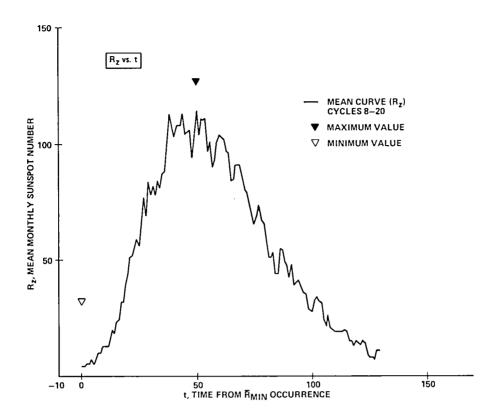


Figure 13. Mean monthly sunspot number (based on mean of cycles 8 through 20) versus time from cycle minimum occurrence.

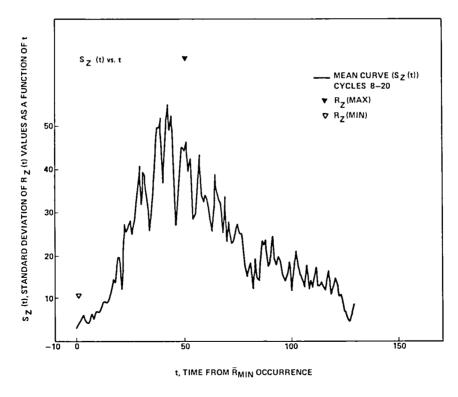


Figure 14. Standard deviation of monthly mean sunspot values versus time from cycle minimum occurrence.

accessible and timely sunspot activity parameters, one can use these curves for comparing individual cycle curves with the mean curve and estimating near-term sunspot activity (e.g., the decline of cycle 21 in terms of R_Z). Appendix B contains figures showing each cycle versus the mean curve in terms of R_Z (t).

Similarly, Figure 15 displays the mean $\overline{R}_{13}(t)$ curve. It is much smoother in appearance than the $R_Z(t)$ curve. Again, minimum and maximum values are identified by light and dark triangles, respectively. It is seen that $\overline{R}_{13}(t=0)$ corresponds to \overline{R}_{MIN} and has a value of about 5.2. \overline{R}_{13} is at maximum at t=48; its value is about 106.9 (about 9.3 units less than that determined by the schematic approach). Figure 16 plots the standard deviation of $\overline{R}_{13}(t)$ value as a function of t; i.e., $t_{13}(t)$. It too is a much smoother curve than its counterpart in t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum t=1000 is about 2.7 and 3.3 is at maximum at the 2.0 is at 2.0 is

Table 5 contrasts nine parameters obtained using the schematic curve approach with that using the mean curve approach. Very little difference is noted (with the possible exception of \overline{R}_{MAX} and \overline{R}_{CHM} being about 8 percent lower, and ASC_{CHM} being about 13 percent lower; D_{CHM} is about 5 percent higher), so either approach can be useful when comparing a particular cycle with parametric means or mean cycle values to estimate near-term sunspot activity within a given cycle.

Figure 17 shows the mean $\overline{R}_{13}(t)$ curve for the subgroups HIGH- \overline{R}_{MAX} cycles (top curve) and LOW- \overline{R}_{MAX} cycles (bottom curve). The minima and maxima are again identified using light and dark

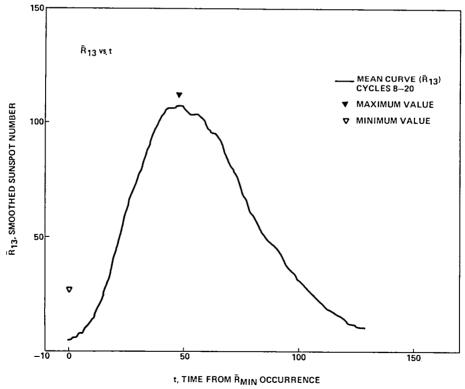


Figure 15. Smoothed sunspot number (based on mean of cycles 8 through 20) versus time from cycle minimum occurrence.

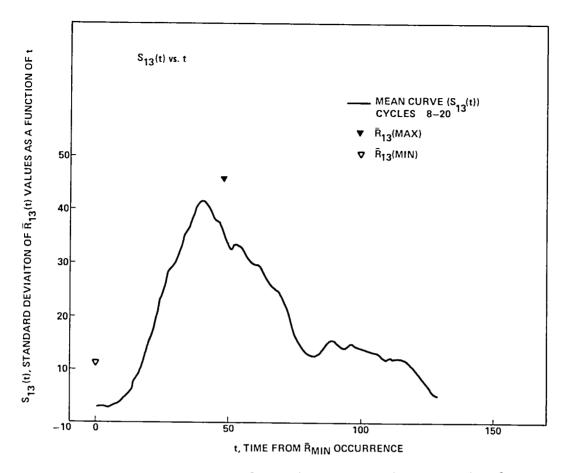


Figure 16. Standard deviation of smoothed sunspot values versus time from cycle minimum occurrence.

TABLE 5. COMPARISON OF SCHEMATIC AND MEAN CURVE PARAMETRIC VALUES FOR SELECTED PARAMETERS (BASED ON CYCLES 8 THROUGH 20)

PARAMETER	SCHEMATIC CURVE VALUE	MEAN CURVE VALUE
R _{MIN}	5.18	5.18
\overline{R}_{MAX}	116.18	106.87
R _{CHM}	60.72	56.03
ASC _{CHM}	26.31 (24.83)*	~23
D _{CHM}	56.08	~59
ASC	48.15	48
DES	83.46	82
MIN-MIN PERIOD	131.62	130
$^{\Phi}$ MAX	0.367	0.369

^{*} VALUE EXCLUDING SCN 9

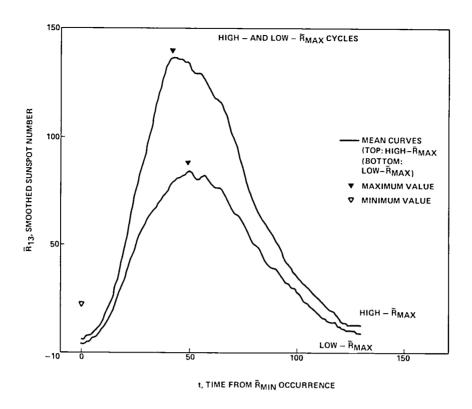


Figure 17. Smoothed sunspot number for HIGH- and LOW- \overline{R}_{MAX} cycles versus time from cycle minimum occurrence.

triangles, respectively. For HIGH- \overline{R}_{MAX} cycles, \overline{R}_{MIN} is about 6.3 and \overline{R}_{MAX} , occurring at t = 42, is about 136.1. For LOW- \overline{R}_{MAX} cycles, \overline{R}_{MIN} is about 4.3 and \overline{R}_{MAX} , occurring at t = 49, is about 83.8. Thus HIGH- \overline{R}_{MAX} cycles tend to have shorter ascent periods than LOW- \overline{R}_{MAX} cycles.

Table 6 contrasts the same nine parameters used in Table 5 obtained using the schematic HIGH- and LOW- \overline{R}_{MAX} cycle curves (Table 3) with that using the mean HIGH- and LOW- \overline{R}_{MAX} cycle curves (shown in Fig. 17). Again, little difference is observed between parameter mean values; the mean curve values tend to run a little lower than the schematic curve values (except for D_{CHM} HIGH- \overline{R}_{MAX} cycles which run higher). (Table 6 does not provide a comparison with DES, MIN-MIN PERIOD and Φ_{MAX} , because the calculation was performed to deduce the mean curve for only 130 months, for comparison to the mean cycle curve shown in Figure 15.) Appendix D contains figures showing each cycle versus the mean curves in terms of \overline{R}_{13} (t) for the subgroups HIGH- and LOW- \overline{R}_{MAX} cycles.

Figure 18 gives the mean $\overline{R}_{13}(t)$ curve for the subgroups LONG-PERIOD cycles (the lower \overline{R}_{13} curve) and SHORT-PERIOD cycles (the higher \overline{R}_{13} curve). The minima and maxima are again identified using light and dark triangles, respectively. For LONG-PERIOD cycles, \overline{R}_{MIN} is about 5.5 and \overline{R}_{MAX} , occurring at t=50, is about 91.8. For SHORT-PERIOD cycles, \overline{R}_{MIN} is about 4.8 and \overline{R}_{MAX} , occurring at t=44, is about 127.1. Thus, LONG-PERIOD cycles, averaging about 18 months longer than SHORT-PERIOD cycles, tend to be lower in \overline{R}_{MAX} value (on average, about 35 units smaller), and longer in both ASC and DES periods (on average, about 6 and 12 months, respectively).

TABLE 6. COMPARISON OF SCHEMATIC AND MEAN CURVE PARAMETRIC VALUES FOR SELECTED PARAMETERS (BASED ON HIGH- AND LOW- \overline{R}_{MAX} CYCLE GROUPINGS)

	HIGH-	-R _{MAX}	LOW-	-R _{MAX}
PARAMETER	SCHEMATIC VALUE	MEAN CURVE VALUE	SCHEMATIC VALUE	MEAN CURVE VALUE
R _{MIN}	6.27	6.27	4.26	4.26
\overline{R}_{MAX}	148.62	136.07	88.39	83.76
R _{CHM}	77.48	71.17	46.36	44.01
ASC _{CHM}	29.00	~24	24.00	~23
D _{CHM}	50.00	~55	61.29	~61
ASC	44.17	42	51.57	49
DES*	85.67	-	81.57	-
MIN-MIN PERIOD*	129.83	_	133.14	-
$^{\Phi}$ MAX *	0.340	-	0.390	_

^{*}CALCULATION PERFORMED FOR ONLY 130 MONTH PERIOD, FOR COMPARISON TO MEAN CYCLE CURVE.

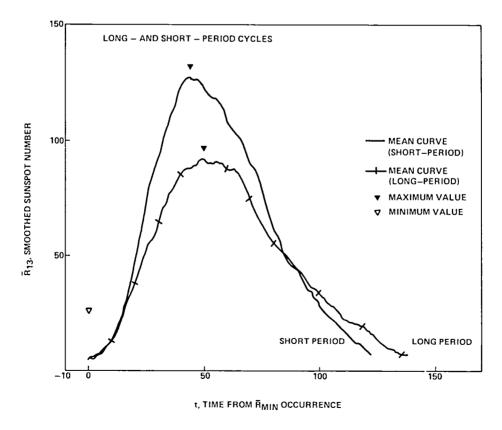


Figure 18. Smoothed sunspot number for LONG- and SHORT-period cycles versus time from cycle minimum occurrence.

Table 7 (as did Tables 5 and 6) contrasts the nine aforementioned parameters between schematically determined values and those determined by means of the mean curves (shown in Fig. 18). Slight differences are again noted. An interesting finding is that, regardless of grouping, ASC_{CHM} is, on average, always about 23 months. Thus, one could estimate \overline{R}_{MAX} and, perhaps, MIN-MIN PERIOD some 2 years or so prior to \overline{R}_{MAX} occurrence, based on a measure of \overline{R}_{13} at ASC_{CHM} (since this corresponds to \overline{R}_{CHM} and $\overline{R}_{MAX} = 2$ $\overline{R}_{CHM} - \overline{R}_{MIN}$; the calculated \overline{R}_{MAX} would then be either above/equal to the mean or below the mean \overline{R}_{MAX} value and, consequently, an estimation of MIN-MIN PERIOD could be made, based on Table 3, for example). Appendix E contains figures showing cycle versus the mean curves in terms of $\overline{R}_{13}(t)$ for the subgroups LONG- and SHORT-PERIOD cycles.

TABLE 7. COMPARISON OF SCHEMATIC AND MEAN CURVE PARAMETRIC VALUES FOR SELECTED PARAMETERS (BASED ON LONG- AND SHORT-PERIOD CYCLE GROUPINGS)

LONG-PERIOD

SHORT-PERIOD

PARAMETER	SCHEMATIC VALUE	MEAN CURVE VALUE	SCHEMATIC VALUE	MEAN CURVE VALUE
\overline{R}_{MIN}	5.49	5.49	4.83	4.83
R _{MAX}	101.10	91.80	133.78	127.10
R _{CHM}	53.33	48.65	69.35	65.97
ASC _{CHM}	27.00	~23	25.50	~23
D _{CHM}	57.86	~63	54.00	~56
ASC	50.14	50	45.83	44
DES	89.86	90	76.00	78
MIN-MIN PERIOD	140.00	140	121.83	122
^Ф МАХ	0.359	0.357	0.376	0.361

Table 8 lists R_Z , \overline{R}_{13} , s_Z , and s_{13} values, as a function of t, for the mean cycle curves (cycles 8 through 20) shown in Figures 13 through 16, and \overline{R}_{13} and s_{13} values for the mean of HIGH- \overline{R}_{MAX} , LOW- \overline{R}_{MAX} , LONG-PERIOD, and SHORT-PERIOD cycles, shown in Figures 17 and 18. One can use this table, for example, to compare R_Z and \overline{R}_{13} values of a particular cycle to the various means and, perhaps, determine near-term sunspot activity levels and the group in which a particular cycle might belong.

E. Linear Regression Equations Based on SCN

Recalling Table 2 (and Fig. 9), one can plot \overline{R}_{MIN} , \overline{R}_{MAX} , ASC, DES, and \overline{R}_{MIN} (SCN + 1), along with two other parameters – SLOPE_{ASC} and SLOPE_{DES} – versus SCN. This has been done in Figure 19. (Table 9 lists those data.) \overline{R}_{MIN} and \overline{R}_{MAX} show a definite downward trend for cycles 8 through 14 and an upward trend for cycles 14 through 20. ASC shows the converse of this; i.e., an upward trend for cycles 8 through 14 and a downward trend for cycles 14 through 20. The trends have been graphically illustrated by the use of linear regression lines shown in Figure 19.

TABLE 8. MEAN SMOOTHED SUNSPOT NUMBER VALUES VERSUS t FOR SELECTED CYCLE GROUPINGS

		MEAN OF CYC	LES 8-20		MEAN O			OF LOW- CYCLES	MEAN OF			F LONG- CYCLES
t	Rz	Sz(t)	R 13	S 13 ^(t)	R 13	S ₁₃ (t)	R 13	S 13 ^(t)	Ř 13	S (t)	R 13	S (t)
0	4.08	2.99	5.18	2.74	6.27	2.52	4.26	2.57	4.83	2.23	5.49	3.07
1	4.33	4.23	5.33	2.81	6.43	2.54	4.39	2.68	4.93	2.23	5.67	3.19
2	4.78	5.10	5.83	2.92	6.97	2.56	4.86	2.86	5.3 8	2.29	6,21	3.33
3 4	5.38 6.65	6.11 5.03	6. 4 5 7.07	2.88	7.65	2.38 2.05	5.43 E 90	2.87 2.78	5.93	2.03	6.90	3.38
5	5.15	4.22	7.65	2.74 2.69	8.33 8.93	1.72	5.99 6.54 7.37	2.70 2.87	6.65 7.45	1.83 1.96	7.43 7.81	3.28 3.17
6	6.58	4.15	8.47	2.69 2.93 3.21 3.41	9.75	1.88	7.37	2.87 3.21 3.43	8.40	2.29	8.53	3.38
ž	10.32	6.09	9.62	3.21	11.03	2.24	8.41	3.43	9.55	2.82	9.69	3.52
8	10.18	5.08	10.82	3.41	12.48	2.56	9.39	3.41	10.87	3.20	10.77	3.58
9	12.88	6.81	12.12	3.71	13.98	2.91	10.53	3.57	12.17	3.59	12.09	3.80
10	12.88	6.60	13.60	4.12	15.57	3.34	11.91	3.97	13.65	4.01	13.56	4.21
11	12.58	7.11	15.43	4.71	17.65	4.03 4.73	13.53	4.41	15.77	4.69	15.14	4.71
12	15.75	8.99	17.59	5.31	20.15	4.73	15.40	4.77	18.02	5.40	17.23	5.21 5.80
13 14	19.92 18.22	9.25 9.12	19.85 22.46	6.00 7.19	22.72	5.78 7.42	17.39 19.47	5.01	20.27	6.21 7.68	19.49	5.80
15	23.35	9.28	25.45	8.11	25.95 39.53	8.39	21.97	5.44 E.0E	23.18 26.33	7.68 9.65	21.84 24.70	6.69 7.32
16	23.98	11.10	28.70	8.97	29.52 33.38	9.47	24.69	5.95 6.14	20.33	8.85 10.32	27.81	7.32 7.51
17	31.89	14.60	32 16	10.48	37.58	11.72	27.51	6.27	29.73 33.78	12.71	30.77	7.82
18	31.72	14.04	32.16 35.75	12.06	41.90	13.83	30.49	6.78	38.45	14.87	33.44	8.30
19	39.41	19.65	39.05	13.58	45.93	15.66	33.16	7.56	43.25	16.68	35.46	8.72
20	43.56	19.67	42.55 46.74	15.10 16.71	50.32	17.32	35.90	8.38	47.92 52.83 57.92	18.23	37. 9 6	9.63
21	51,36	12.29	46.74	16.71	55.50	19.11	39.23	9.12	52.83	19.87	41.51	10.99
22 [.] 23	52.47	27,31 25.72	50.89 54.95	18.60 20.72	60.60	21.48	42.57	9.84	57.92	22.15	44.87	11.97
23 24	55.77 58.74	25.72 26.56	54.95 59.05	20.72 22.58	65.95	23.97 26.05	45.53 48.59	10.58 11.33	63.37 69.05	24.64	47.74	12.80
25 25	56.28	28.27	62.78	24.26	71.27 75.97	28.27	48.59 51.47	11.45	74.42	26.46 28.34	50.49	13.71
26 26	65.67	25.17	66.02	26.11	80.17	30.94	53.90	11.25	7 9.2 3	30.69	52.80 54.70	13.75
27	76.72	28.03	68.82	27.92	84.07	33.26	55.74	11.36	83.27	32.91	54.70 56.43	13.43 13.67
28	69.47	34.61	71.38	28.76	87.08	34.09	57.93	12.14	86.45	33.54	58.47	14.50
29	84.18	41.03	73.88	29.06	90.10	33.90	59.97	12.81	89.27	33.45	60.69	15.23
30	77.99	32.11	76.42	30.00	94.20	34.20	61.19	12.88	92.02	34.59	63.06	16.09
31	82.48 78.28	39.42	79.38	31.21	98.87 104.75	34.82	62.69	12.87	94.75	36.28	66.21	17.46
32 22	76.26 84,22	38.82 32.83	83.10 86.39	32. 8 5 34.75	110.35	35.48 36.35	64.54 65.86	13.39 14.01	98.50 103.75	38.65 41.09	69.90	18.62 19.22
32 33 34 35 36	81.10	26.04	89.09	35. 78	114.68	36.43	67.16	14.06	102.75 106.32	42.00	72.37 74.33	19.22 19.85
35	87.06	32.46	91.48	36.38	118.25	36.27	6 8 -53	13.74	108.83	42.47	76.60	20.79
36	88.31	42.69	93.77	37.84	121.87	37.71	69.69	13.54	111.62	44.22	78.47	21.82
37	98.13	49.95	96.04	39.22	125,18	38.97	71.06	14.24	114.38	45.33	80.31	21.82 23.67
38	113,14	50.05	98.57	40.52	128.95	39 .59	72.53	15.38	117.45	45. 9 5	82.39	25.91
39	108.06	52.10	100.87	41.42	132.02	40.31	74.17	15.79	120.50	46.35	84.04	27.04
40	103.02	37.33	102.67	41.48	133:93 135.63	40.50	75.87	15.31	123.02	45.90	85.23	27.04
41	107.46 108.34	51.31 55.33	104,47 105,50	41.26	135.63 136.08	40.16 3 9 .94	77.76	15.25	124.92 125.93	45.53	86.94	26.80
42 43	108,34	49.32	105.50	40.90 40.12	136.07	39.94 38.72	79.29 80.17	15.81 16.18	125.93 126.65	44.86	87.99 8 8. 24	26.77 26.57
44	113.28	52.52	106.22	39.11	135.88	36.82	80.17 80.79	16.18	127.10	43.23 41.40	88.31	26.03
45	103.93	39.92	106.07	37.92	135.07	34.88	81.21	17.01	126.28	40.48	88.74	24.82
46	104.91	27.25	106.22	37.48	135.07	33.89	81.50	17.34	125.75	40.81	89.49	23.97
47	106,36	34.32	106.65	37.30	135.07	33.23	82.30	18.75	125.95	40.75	90.11	23.81
48	93.88	41.66	106.87	35.96	134.32	30.73	83.34	19.84	125.37	39.18	91.01	23.27
49	103.75	45.34	106.52	34.29	133.08	28.08	83.76	19.63	123,77	37.49	91.74	22.49

TABLE 8. (Continued)

		MEAN OF CYC			MEAN O			OF LOW— CYCLES	MEAN OF PERIOD O		PERIOD	F LONG- CYCLES
t	Rz	SZ (t)	R ₁₃	S 13 ^(t)	R 13	S (t)	R 13	S 13 ^(t)	R 13	S (t)	₹ 13	S (t)
50	113.52	44,54	105,55	32,86 32,52	131,07	26,48 26,26 26,46 26,31 26,73 28,25 29,31	83,67	19,17	121,58	36,05	91,80	21,95
51	103,77	46.38	104,45	32,52	129,50	26.26	82,97	19.33	119,90	35,47	91,20	22.47
52	111.02	39,89	103,40 102,75	33,10	129,02	26.46	81.44	19.77	118,82	35,70	90.19	23,76
53	109,76	39,89 42,36 28,71	102.75	33,10 33,28 33,12 32,75 32,08 30,93	129,07	26,31	82,97 81,44 80,19 80,30 81,27	19,01	118.02	35,56	8 9.6 6	24,53
54	111.36	29.93	10 2. 95 10 3. 37	33,12 22.75	129,38	26,73	80,30	17,67	117,90	35,26	90,14 90,84 90,54	24,81
55	97 . 05 101 . 18	29,53 3E 40	103,37	34,/5	129,15	28,25	81.27 81.66	15,81	117,98	35,55	90,84	23,86
56	89,54	35,48 43,58	102,88 102,09	32,00 20.02	127,63 125,88	28.24	81,70	14,12	117.27	35,93	90,54	21,80
57	94,32	34,02	101,20	30.07	124,05	27 .4 7	80.90	19.77 19.01 17.67 15.81 14.12 13.64 14.51	116,30 113,85	34.76	89.91	20,34
58 59	100,99	32,71	99,52	29,93	122,00	27.47 27.64	80.24	14,30	111.95	3 4.9 1	89.64	20.03
										33,93	88,86	23,76 24,53 24,81 23,86 21,80 20,34 20,03 20,77
60	104.35	34,12	97 . 35 96 . 02	29,70 29,59 29,16 28,25 27,38 26,47 25,68 25,15 24,90 24,67	119,73 118,33 117,43	27,16 26,76 26,22 24,58 22,70 20,94 18,76 16,95 16,20	78.17	14,45 14,83 14,53 14,17 13,89 14,22 14,99 15,85	108,42	33,42 32,99	87,87	22,04 23,02 23,10 22,46 21,91 21,82 21,92
61	103.46	31,86	96,02	29,59	118,33	26,76	76,89	14,83	106,15	32,99	87,33	23,02
62	101.78	28,76	95,32	29,16	117.43	26,22	76.89 76.37 76.19 75.93	14,53	104,78	32,45	87,21	23,10
63	96,85	26.17	94.95	28,25	116,85	24,58	76.19	14.17	103.65	31,62	87,50	22,46
64	96,15	28.76 26.17 38.71 36.92 33.38	95,32 94,95 94,63 93,38	27.38	116,85 116,45 114,70	22,70	75.93	13,89	102.98	30.58	87,33 87,21 87,50 87,47	21,91
65	84,28	36,92	93,38	26,47	114,70	20,94	75,10	14,22	101,93	28,45	86.04	21,82
66	85.31	33,38	91,41	25,68	112,35	18.76	73.46 71.36	14,99	100,08	26,98	83,97	21,92
67	90.85	32.61 25.50 33.64	89,04	25,15	109.67	16,95	/1,36	15,85	97,55	25,88	81.74	22,03 22,33 21,96
68	90.98 90.76	23.50	86.41 83.72	24,90	106,50 103,33	16,20 16,40	69,19 66.91	16,59 16,95	94.55	25.26	79,43	22,33
69	94.76								91.88	25,15	76.73	2 1,9 6
70	85.21	23,55 27,92	81 . 32 79 . 71	24.01 22.74	100,43	16 . 24 15 . 45	64 . 93 64 . 03	16.21 14.94 14.03	89,90	24.22	73.96	21.22
71	79,83	27.92	79.71	22,74	98,00	15.45	6 4.0 3	14.94	88,53	22.67	73,96 72,14	19.90
72	78.48	23,38 23,88	78.44 76.58 74.20 71.25 68.17 65.62	21,50 19,94 18,32	95.67	14.85	63,67 62,94	14.03	87,98	21,37	70.76	21,22 19,90 17,94
73	72,82	23,88	76,58	19,94	92,48	14,36	62 ₄ 94	12,59 11,02	86,23	20.05	68.30	15.66 13.95 13.21 12.69 12.06
74	68.92	25.98	74,20	18.32	88.65	14.07	61.81	11,02	83,25	18,66	66,44	13.95
75	65,45	27.03 25.33 25.42	71,25	16,91 15,58 14,82	84 , 30	13.24 11.97 11.34	60,06	10.47	79,58	16,94	64.10	13.21
76	69,48	25,33	68,17	15.58	80.02	11,97	58.01 55.89	10,22	75,37	15.55	62,00	12,69
77	72,52	25,42	65,62	14,82	76.98	11.34	55,89	9,60	71,68	15,42	60.43	12.06
78	66,82	18.69 15.52	63.08 60.45	14.16	74,02	10,75 10,29	53,71	10,47 10,22 9,60 8,95 8,79	68.32	15,44	68.30 66.44 64.10 62.00 60.43 58.60	11,17
79	64,61	15,52	60.45	13,50	70.80	10,29	51,59	8,79	64,87	14,95	56,67	10.76
80	60,25	17.24	58,66	13,05	68,38 66,87 65,25 62,98 60,87 59,68	9,79 9,57 9,17 8,46 7,77 7,81 8,56 9,27	50 . 33 49 . 69	9.14 9.31 9.71 10 . 85	62,10	14,46	55,71	10,87
81	50,63	18.62	57,62	12.74	66,87	9,57	49.69	9.31	60,40	14,00	55,23	11,00
82	51,11	12,40 19,32	56,33	12,56	65,25	9,17	48.69	9.71	58.57	13,86	54,41	10,98
83	52,84	19.32	56.33 54.38 52.27 50.58	12.65	62,98	8.46	47,00	10,85	55,88	13.96	54,41 53,09 51,54	11,00 10,98 11,24 11,68 12,62 14,02
84	44,42	14,93 14,29	54,27	12,85 13,38 14,22	60,87	7.77	44.90 42.77	11,71 12,17 13,27	53,12	1 4. 04 1 4. 18	51.54	11,68
85	43.88	14,29	50,58	13,38	59.68	/ ₂ 81	44//	12,17	51,33	14.18	49.93 48.46	12,62
86	54.92	23.47 22.88	49.03 47.73	14,22	58,30 56,73	8,56	41,09 40.01	13-27	49.70	14,43	48.46	14,02
87	54,43 49,01	22,88	47.73 46 . 88	14.67	20./3	9,27	40.01	14.03	48.22	14,26	47,31 46,46	15,00 15,79 16,71
88 89	46,54	23.82 17.61	45,85	15.07 15.55	55,60	10,28 11,23	39,41 38,91	14,49 15,38	47.38	14,16	46.46	15,79
89	46,54				53,95			15,36	46.10	14,08	45,64	16,71
90	42.37	18,27	44,88 44,12 42,71 40,55 38,52 36,99 36,08 35,05 34,05	15,64 15,37 14,85	52,23	11,63 11,53 11,08	38,59 38,21 37,06 35,29 33,73 32,41 31,60 30,89	15.88 15.79 15.33 15.27 15.20 14.65 14.52 14.55 13.99 13.32	44.77	13,44	44.99	17 ,3 0 17 , 31
91	48.46	24,61 19,39	4% 12 42 71	10,37	51.02	11,03	30,∡1 27.00	15./9	43.70	12,73	44,49 43,20	17,31
92 93	39.08	19.39	42.7 I	14,00	49.30	11,06	37,00	15,33	42.13	11.77	43,20	17,03 17,11
93 94	40,48	18.41 20.02	40,55 20 K2	14,46 14,30 14,34 14,65 14,75 14,49	46.70	10,53	33 . ∠3	15,27	39.43	10,45	41.51	17,11
94 95	41,27	ZU,UZ	30,52	14,30	44.12	10,/3	33,/3 22.41	13,∡U 14 €E	36.85	9,35	39.96	17.33
96	38.07 35.56	18.61	36 00 30,33	14.54 14.65	42,33	11.31	34.41 21.60	14.00	35.43	8.99	38,33 37,34	17,56
96 97		15,47 14,54 15,34	36,06	14.00	41,30	12.44	30.00	14,DZ 14.55	34,60 33,47	9.35	37,34	17,89
97 98	34,81 29,81	15.24	35,05 34.05	14.49	39,3∡ 29,46	13.44	30,03 30 27	14,00	33,47 22, 25	10,05	36,41 35,50	17.71
99	27 .9 8	18,43	33,02	14,19	39.92 38.45 37.03	10,53 10,73 11,91 12,98 13,44 13,80 14,11	30,27 29,57	12.22	32,35 30,90	10.53	<i>\$</i> 3,50	17,33 17,56 17,89 17,71 17,04
33	27.50	10,73	33,02	17,10	37,03	14,11	23,37	13,34	30,30	10,10	34,83	16,71

TABLE 8. (Concluded)

		MEAN OF CYC	LES 8-20		MEAN O		Rassy	OF LOW- CYCLES	MEAN OF SHORT— PERIOD CYCLES		PERIOD	F LONG- CYCLES
t	Rz	S _Z (t)	R 13	S (t)	R 13	S (t) 13	R 13	S 13 ^(t)	A ₁₃	S (t)	R 13	S (t)
100 101 102 103 104 105 106 107 108	26.58 32.48 34.22 32.24 31.21 23.50 21.05 26.22 21.32 19.81	12.18 18.41 21.44 17.91 15.92 15.13 13.31 17.89 12.81	31.46 30.12 28.05 27.83 26.78 25.92 25.22 24.37 23.25 22.07	13.92 13.64 13.55 13.46 13.28 13.05 13.03 12.89 12.41 11.96	35.43 33.98 32.92 31.77 30.63 29.78 29.12 28.08 26.72 25.53	14.12 13.65 13.34 13.40 13.12 13.12 13.54 13.46 12.94 12.39	28.06 26.81 25.73 24.46 23.47 22.61 21.89 21.19 20.27 18.10	12.80 12.74 12.82 12.57 12.49 12.03 11.58 11.46 11.10	28.98 27.45 26.12 24.82 23.90 23.13 22.32 21.25 20.12 19.08	9.25 8.13 7.14 6.69 6.76 6.87 6.86 6.66 6.22	33.59 32.41 31.56 30.41 29.24 28.31 27.71 27.04 25.93	16.63 16.67 16.83 16.84 16.58 16.23 16.17 15.97
110 111 112 113 114 115 116 117 118	19.28 18.55 19.28 18.95 20.19 18.62 15.48 15.21 13.12	13.06 17.43 13.40 13.22 13.80 12.77 12.44 16.48 11.25 12.62	20.84 19.84 19.15 18.36 17.56 17.02 16.55 16.17 15.56 14.67	11.84 12.07 12.13 11.96 11.93 12.13 12.09 11.75 11.25 10.82	24.50 23.60 22.67 21.70 20.70 20.08 19.45 18.78 17.97 16.82	11.93 11.61 11.34 11.41 11.86 12.15 12.18 11.81 11.20 10.72	17.70 16.61 16.14 15.50 14.87 14.40 14.06 13.93 13.50 12.39	10.82 11.51 11.97 11.59 11.30 11.48 11.44 11.21 10.87	17.97 16.97 16.02 14.97 13.98 13.12 12.22 11.60 11.08 10.13	5.87 6.10 6.50 6.55 6.75 6.77 6.24 5.43 4.79 4.44 3.98	24.63 23.30 22.30 21.84 21.27 20.63 20.37 20.26 20.09 19.40 18.56	14.89 14.68 14.88 14.95 14.36 14.30 14.68 14.71 14.26 13.64 13.08
120 121 122 123 124 125 126 127 128 129	14,06 13,37 14,78 13,72 9,39 7,91 8,38 6,83 11,84 12,15	14.80 14.13 10.64 10.83 7.50 7.07 5.26 4.97 6.41 8.59	13.72 12.75 12.10 11.82 11.65 11.52 11.28 11.05 10.75	10.27 9.65 9.04 8.31 7.76 7.27 6.44 5.56 5.16 5.08	15.68 14.40 13.62 13.48 13.15 13.00 12.95 12.67 12.55 12.87	10.31 10.01 9.72 9.13 8.62 8.12 7.22 6.18 5.65 5.23	12.04 11.33 10.80 10.40 10.37 10.26 9.86 9.66 9.21 8.70	9.93 9.10 8.18 7.23 6.68 6.18 5.29 4.51 4.11	9.02 7.92 7.32	3.52 3.26 3.30	17.76 16.89 16.20 15.54 14.86 14.14 13.14 12.11 10.99 9.91	12.24 11.26 10.29 9.38 8.98 8.68 7.93 6.96 6.40 6.01
130 131 132 133 134 135 136 137 138 139											9.24 8.90 8.64 8.36 7.94 7.41 7.00 6.76 6.73 6.83	5.74 5.89 6.12 6.30 6.43 6.10 5.35 4.85 4.63 4.34

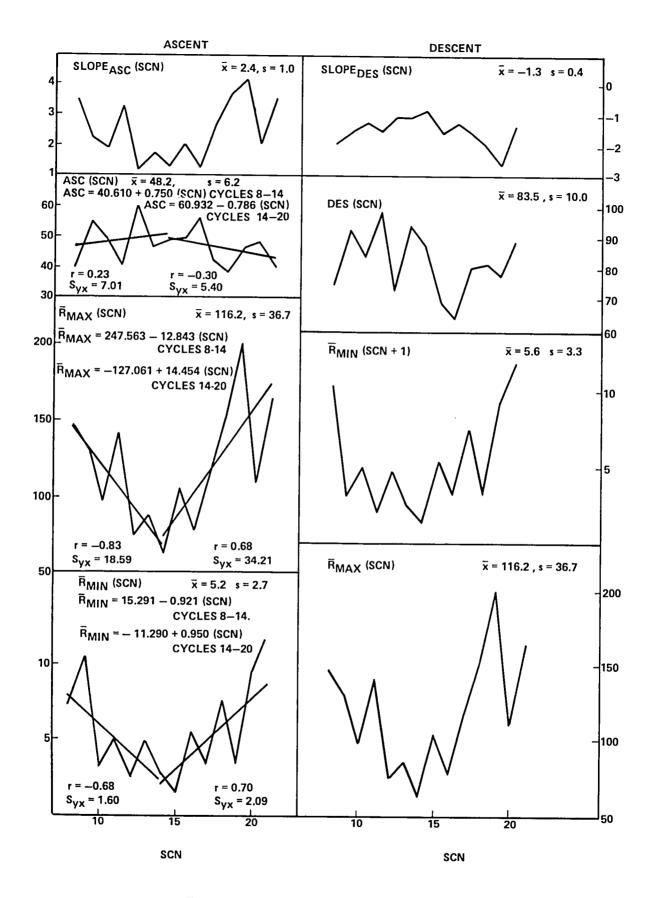


Figure 19. Selected parameters versus SCN.

TABLE 9. LIST OF SELECTED PARAMETRIC DATA FOR SCN 8 THROUGH 21

SCN	\overline{R}_{MIN}	\overline{R}_{MAX}	ASC	SLOPEASC	DES	R _{MIN} (SCN + 1)	SLOPEDES
8	7.3	146.9	40	3.490	76	10.5	-1.795
9	10.5	132.0	55	2.209	94	3.2	-1.370
10	3.2	97.9	50	1.894	85	5.2	-1.091
11	5.2	140.5	41	3.300	100	2.2	-1.383
12	2.2	74.6	60	1.207	74	5.0	-0.941
13	5.0	87.9	47	1.764	96	2.7	-0.888
14	2.7	64.2	49	1.255	89	1.5	-0.704
15	1.5	105.4	49	2.120	71	5.6	-1.406
16	5.6	78.1	57	1.272	65	3.5	-1.148
17	3.5	119.2	43	2.691	82	7.7	-1.360
18	7.7	151.8	39	3.695	83	3.4	-1.788
19	3.4	201.3	47	4.211	79	9.6	-2.427
20	9.6	110.6	49	2.061	91	12.2	-1.081
21	12.2	164.5	42	3.626	_	_	_

For $\overline{R}_{\mbox{MIN}}$, the regression lines are given as

CYCLES 8-14:
$$\overline{R}_{MIN} = 15.291 - 0.921 \text{ (SCN)}$$
 (19a)

and

CYCLES 14-20:
$$\overline{R}_{MIN} = -11.290 + 0.950 \text{ (SCN)}$$
 (19b)

The Pearson correlation coefficient r equals -0.68 for cycles 8 through 14 and the standard error of estimate S_{yx} equals 1.60; for cycles 14 through 20, r = 0.70 and $S_{yx} = 2.09$. ($\overline{x} = 5.2$ and s = 2.7 for the entire data set.)

For $\overline{\boldsymbol{R}}_{\mbox{\scriptsize MAX}},$ the regression lines are given as

CYCLES 8-14:
$$\bar{R}_{MAX} = 247.563 - 12.843$$
 (SCN) (20a)

CYCLES 14-20:
$$\overline{R}_{MAX} = -127.061 + 14.454 \text{ (SCN)}$$
 (20b)

For cycles 8 through 14 r = -0.83 and S_{yx} = 18.59, and for cycles 14 through 20 r = 0.68 and S_{yx} = 34.21. (\overline{x} = 116.2 and s = 36.7 for the entire data set.)

For ASC, the regression lines are given as

CYCLES 8-14:
$$ASC = 40.610 + 0.750 (SCN)$$
 (21a)

and

CYCLES 14-20:
$$ASC = 60.932 - 0.786 (SCN)$$
 (21b)

For cycles 8 through 14 r = 0.23 and S_{yx} = 7.01; for cycles 14 through 20 r = -0.30 and S_{yx} = 5.40. (\overline{x} = 48.2 and s = 6.2 for the entire data set.)

The terms SLOPEASC and SLOPEDES are simply defined as

$$SLOPE_{ASC} = (\overline{R}_{MAX} - \overline{R}_{MIN})/ASC$$
 (22a)

and

$$SLOPE_{DES} = [\overline{R}_{MIN}(SCN + 1) - \overline{R}_{MAX}]/DES \qquad (22b)$$

A regression fit between $SLOPE_{ASC}$ and $SLOPE_{DES}$ is shown in Figure 20. The regression line is deduced as

$$SLOPE_{DES} = -0.335 - 0.418 SLOPE_{ASC} . (23)$$

The Pearson r equals -0.91 and S_{yx} equals 0.20. (From Fig. 19, it is seen that $\overline{x} = 2.4$ and s = 1.0.)

In Figure 21 DES has been plotted against ASC. At first glance, the correlation looks very poor. However, if one tags each data point as being either a long-period or short-period cycle, one observes fairly strong correlations. In Figure 21 short-period cycles are identified as the lower diagonal line (triangles) and the long-period cycles as the upper diagonal line (circles). Regression equations for these groupings are given below as:

LONG-PERIOD CYCLES: DES =
$$147.170 - 1.143$$
 ASC (24a)

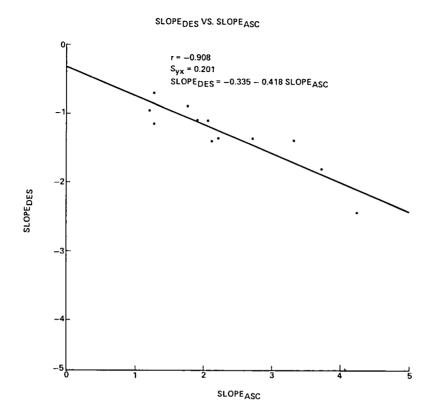


Figure 20. Descent slope versus ascent slope.

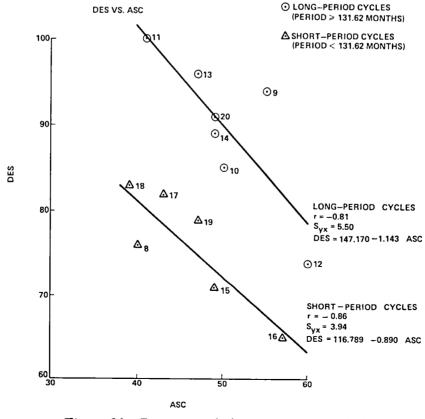


Figure 21. Descent period versus ascent period.

SHORT-PERIOD CYCLES: DES =
$$116.789 - 0.890$$
 ASC . (24b)

For long-period cycles, r = -0.81 and $S_{yx} = 5.50$; for short-period cycles, r = -0.86 and $S_{yx} = 3.94$. (From Fig. 19, it is seen that, for DES, $\overline{x} = 83.5$ and s = 10.0.) If one were to do the linear-regression analysis discarding the concept of long- and short-period cycles, one would deduce the relation

$$DES = 107.920 - 0.508 ASC (25)$$

based on cycles 8 through 20. The Pearson r equals -0.32 and S_{yx} equals 10.29.

In Figure 22, \bar{R}_{MAX} has been plotted against ASC. Regression analysis, using the concept of long-and short-period cycles, results in the following equations:

LONG-PERIOD CYCLES:
$$\overline{R}_{MAX} = 193.157 - 1.836 \text{ ASC}$$
 (26a)

and

SHORT-PERIOD CYCLES:
$$\overline{R}_{MAX} = 295.056 - 3.519 \text{ ASC}$$
 (26b)

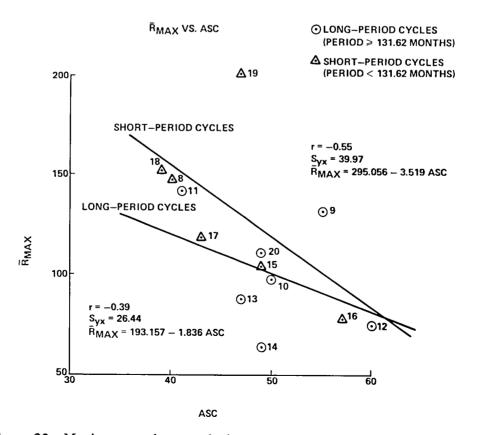


Figure 22. Maximum cycle smoothed sunspot number versus ascent period.

For long-period cycles, r = -0.39 and $S_{yx} = 26.44$; for short-period cycles, r = -0.55 and $S_{yx} = 39.97$. Disregarding class results in

$$\overline{R}_{MAX} = 274.016 - 3.278 \text{ ASC}$$
 , (27)

with r = -0.56 and $S_{yx} = 33.15$, or

$$ASC = 59.071 - 0.094 \,\overline{R}_{MAX} \quad , \tag{28}$$

with r = -0.56 and $S_{VX} = 5.62$.

In Figure 22, it should be noted that cycle 19 looks suspiciously abnormal; it is 1.79 S_{yx} units too high based on equation (26a), 2.45 S_{yx} units too high based on equation (27), and 2.32 s units too high based on the parametric mean (Table 2). So, deleting SCN 19 from the analysis results in

$$\overline{R}_{MAX} = 301.221 - 3.968 \text{ ASC}$$
 (29)

for SHORT-PERIOD CYCLES (r = -0.97 and $S_{yx} = 7.96$) and

$$\overline{R}_{MAX} = 257.411 - 3.074 \text{ ASC}$$
 (30)

for cycles 8 through 20 (of course, deleting SCN 19; r = -0.70 and $S_{VX} = 22.12$).

A comparison of \overline{R}_{MAX} and \overline{R}_{MIN} using the concept of long- and short-period cycles results in the findings (Fig. 23)

LONG-PERIOD CYCLES:
$$\overline{R}_{MAX} = 69.044 + 5.839 \overline{R}_{MIN}$$
 (31a)

and

SHORT-PERIOD CYCLES:
$$\overline{R}_{MAX} = 124.294 + 1.964 \overline{R}_{MIN}$$
 (31b)

For long-period cycles, r = 0.68 and $S_{yx} = 16.64$; for short-period cycles, r = 0.11 and $S_{yx} = 47.59$. Disregarding class results in

$$\overline{R}_{MAX} = 95.596 + 3.862 \overline{R}_{MIN}$$
 , (32)

with r = 0.29 and $S_{VX} = 38.17$. Deleting SCN 19 results in

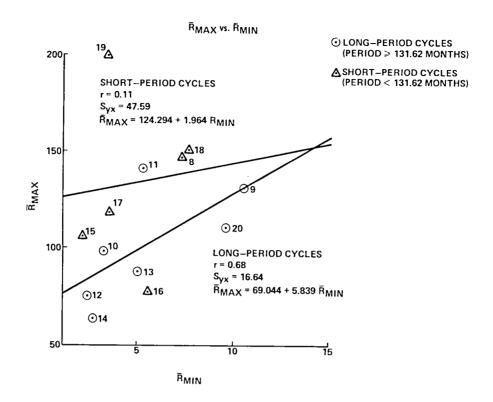


Figure 23. Maximum cycle smoothed sunspot number versus minimum cycle smoothed sunspot number.

$$\overline{R}_{MAX} = 87.594 + 6.384 \overline{R}_{MIN}$$
 (33)

for SHORT-PERIOD CYCLES (r = 0.55 and $S_{yx} = 29.37$) and

$$\overline{R}_{MAX} = 78.411 + 5.756 \,\overline{R}_{MIN}$$
 (34)

for cycles 8 through 20 (minus SCN 19; r = 0.57 and $S_{yx} = 25.51$).

The above equations (and figures) have laid a simple foundation whereby estimations of particularly interesting facets of solar cycle can be made, especially near-term sunspot activity, and perhaps, next cycle activity. The next section will examine more closely correlations between selected individual parameters, other than SCN.

F. Linear Regression Equations Based on Selected Parameters

Table 10 lists values for r, S_{yx} , a_{yx} , and b_{yx} for seventeen parameters (Y variables) correlated with seven selected parameters (X variables) which occur in the early phase of a sunspot cycle and, thus, may have some predictive ability. The X variables are listed vertically in the leftmost column; the Y variables are identified horizontally across the top. For each XY variable combination, four numbers are shown. These numbers are, in descending order, r, S_{yx} , a_{yx} , and b_{yx} . (Recall that linear regressions

TABLE 10. LINEAR REGRESSION COEFFICIENTS FOR CYCLE PARAMETERS BASED ON KNOWN SELECTED EARLY OCCURRING CYCLE PARAMETERS

×	<u>+</u>	R _{MAX}	R _{CHM}	ASC _{CHM}	D _{CHM}	ASC	DES	MIN-MIN PERIOD	MAX- MAX PERIOD	$\Phi_{\sf MAX}$	Φ_{MIN}	tGPV	∆ GPV ^R 13	R ₁₃	^t GNV	GNVR ₁₃	R ₁₃	R _{MIN} (SCN+1)
	r	0.29	0.35	0.43	-0.20	-0.14	0.37	0.28	0.27	-0.31	0.30	0.05	0.31	0.01	-0.36	-0.49	0.54	0.20
	Syx	38.17	19.08	5.70	9.82	6.69	10.09	10.37	13.07	0.05	0.04	10.43	2,34	32.33	7.96	0.86	21.39	3.50
RMIN	аух	95.596	48.138	21.575	59.644	49.740	76.524	126,274	125.419	0.395	0.612	29,132	5.658	60.867	78.384	-4.380	53.867	4.343
	byx	3.862	2.429	0.914	-0.688	-0.307	1.339	1.032	1.226	-0.005	0.004	0.183	0.257	2.435	-1.024	-0.164	4.628	0.235
								ļ										
	r	0.89	0.89	0.25	-0.59	-0.49	-0.05	-0.35	0.22	-0.28	-0.45	0.22		0.75	-0.24	-0.79	0.71	0.32
Δ -	Syx	18.53	9.41	6.12	5.97	5.88	10.84	10.12	13.23	0.05	0.04	10.20		21.37	8.28	0.60	17.91	3.38
[∆] GPV ^R 13	аух	16.048	9.337	21.864	73.010	57.556	84.902	142.458	123.205	0.408	0.685	23.678		4.647	78.804	-3.006	26.576	2.269
	byx	14,321	7.349	0.635	-2.422	-1.345	-0.206	-1.551	1.225	-0.006	-0.007	0.915		9.844	-0.819	-0.318	7.331	0.471
																	- 15	
	r	0.09	0.09	0.69	-0.39	-0.24	0.09	-0.06	0.01	-0.22	-0.05		0.22	0.55	0.04	-0.29	0.15	-0.21
	Syx	39.71	20.33	4.57	9.25	6.55	10.80	10.80	13.57	0.05	0.04		2.41	26.96	8.52	0.95	25.16	3.49
†GPV	ayx	106.404	55.576	13.737	67.270	52.873	80.572	133.425	125.513	0.4001	0.6390		5.462	22.103	72.178	-4.418	66.981	7.726
	byx	0.325	0.171	0.418	-0.372	-0.157	0.096	-0.060	0.208	-0.0011	-0.0002		0.051	1.708	0.030	-0.027	0.361	-0.072
							<u> </u>											
	r		1.00	0.20	-0.50	-0.56	0.10	-0.25	0.21	-0.42	-0.12	0.09	0.89	0.78	-0.34	-0.69	0.88	0.43
_	Syx		1.42	6.19	8.71	5.62	10.80	10.47	13.27	0.05	0.04	10.41	1.15	20,31	8.02	0.72	11.84	3.22
RMAX	аух		1.352	22,708	70.603	59.071	80.439	139.520	123.521	0.4251	0.6446	27.524	0.633	0.170	81.561	-3.255	12,198	1.145
	byx		0.511	0.031	-0.125	-0.094	0.026	-0.068	0.071	0.0005	-0.0001	0.022	0.055	0.631	-0.073	-0.017	0.565	0.038
	r	-0.56	-0.55	0.12	0.21		-0.32	0.31	-0.39	0.83	-0.04	-0.24	-0.49	-0.77	0.53	0.37	-0.65	-0.18
	s_{yx}	33,15	17.01	6.272	9.80		10.29	10,29	12.48	0.03	0.04	10.14	2.15	20.51	7.23	0,92	19.35	3.51
ASC	а _{ух}	274.016	141.082	21.014	40.720		170.920	107.930	169.809	0.078	0.6426	48.136	15.610	251.683	40.964	-7.830	195.470	10.182
	byx	-3.278	-1.669	0.110	0,319		-0.508	0.492	-0.790	0.006	-0.0002	-0.375	-0.179	-3.701	0,667	0.054	-2.443	-0.096
										ļ								
	r	1.00		0.23	-0.50	-0.55	0.12	-0.22	0.22	-0.44	-0.09	0.09	0.89	0.78	-0,36	-0.71	0.90	0.43
_	Syx	2.52		6.15	8.69	5.63	10.77	10.54	13.23	0.05	0.04	10.41	1.14	20.39	7.96	0.70	10.93	3.22
RCHM	аух	-2.103		22,060	71.017	59.262	79.574	138.846	122.783	0.4338	0.6451	27.348	0.488	-1.145	82.188	7.294	9.469	1.006
	byx	1.948		0.070	-0.246	-0.183	0.064	-0.119	0,148	-0.0011	-0.0002	0.045	0.107	1.229	-0.150	-0.034	1.126	0.075
	Ì					<u> </u>												
	7	0.20	0.23		-0.63	0.12	0.27	0.35	0.24	-0.10	0.18	0.69	0.25	0.32	0.03	-0.26	0.34	-0.23
	S _{YX}	39.06	19,88		7.78	6.70	10.44	10,14	13.18	0.05	0.04	7.55	2.39	30.61	8.52	0,96	23.90	3.47
ASCCHM	аух	83.214	41.540		82.495	44.835	71.094	115.939	118.352	0.3880	0.6014	0.008	4.448	30.121	74.106	-4,151	41.611	8.980
,	Ьух	1.253	0.729		-1.004	0.126	0.470	0.596	0.510	-0.0008	0.0012	1.143	0.097	1.648	-0.039	-0.041	1.377	-0.130
										<u> </u>				<u> </u>				

take the form $y = a_{yx} + b_{yx} x$.) All entries in Table 10 are based on cycles 8 through 20; no cycles have been omitted, although in some cases the omission of one or two cycles will greatly improve r and reduce S_{yx} . Since there are 113 linear regressions contained in Table 10, it would be somewhat laborious to discuss each one. Instead, only those of particular interest will be examined.

The first parameter that can be precisely known for a particular sunspot cycle is \overline{R}_{MIN} , since cycles are dated based on the occurrence of this parameter. [Actually, a cycle is already several months underway before the parameter can be calculated; recall its definition based on equation (2).] Thus, one could approximate the seventeen later occurring parameters on the basis of \overline{R}_{MIN} alone. The next most easily observable parameter is $\frac{\Delta}{GPV}$ \overline{R}_{13} . Recall, it is simply the greatest positive value rate of change of \overline{R}_{13} versus time. It usually occurs about 30 ± 10 months into the cycle (although for cycles 19 through 21, it occurred about 22 ± 2 months). Once $\frac{\Delta}{GPV}$ \overline{R}_{13} has been observed, then t_{GPV} is known [and also $\overline{R}_{13}(t_{GPV})$, although it has not been selected as an X variable for any of these regressions]. The final most easily observable early cycle parameter is \overline{R}_{MAX} , which usually occurs about 48 ± 6 months into the cycle. Once it has been observed, then ASC, \overline{R}_{CHM} and ASC_{CHM} are precisely known.

As noted at the beginning of Section III, correlation coefficients (i.e., r) which are large in absolute value (i.e., $|r| \sim 1$) imply a strong relationship between parameters X and Y; on the other hand, correlation coefficients which are small in absolute value (i.e., $|r| \sim 0$) imply very weak relationships, suggesting that parametric mean values and their standard deviations may be the more appropriate statistics to use. A positive correlation (i.e., r > 0) means that as the X variable increases in value, so does Y. A negative correlation (i.e., r < 0) implies an inverse relationship; i.e., as the X variable increases in value, Y decreases. To reduce the number of equations for discussion, $|r| \ge 0.5$ has been arbitrarily selected as the basic criterion for determining when a relatively strong correlation is suggested.

Concerning the X variable \overline{R}_{MIN} , it is positively correlated with about 70 percent of the Y parameters. Five of the correlations are very weak (i.e., |r| < 0.25), eleven are relatively weak (i.e., $0.25 \le |r| < 0.50$), and only one is considered a relatively strong correlation (i.e., $|r| \ge 0.50$). Thus, while approximate values could be predicted for each of the seventeen parameters on the basis of \overline{R}_{MIN} , only the one between \overline{R}_{MIN} and $\overline{R}_{13}(t_{GNV})$ is considered of relative importance. The regression equation associating these two parameters is

$$\overline{R}_{13}(t_{GNV}) = 53.867 + 4.628 \overline{R}_{MIN}$$
 (35)

The Pearson r equals 0.54 and S_{yx} equals 21.39. Since its mean value is 77.84 with a standard deviation of 23.40, one may somewhat more confidently predict it about 6 years prior to its occurrence to an accuracy slightly better than by using the mean value.

Concerning the X variable $\frac{\Delta}{GPV}$ \overline{R}_{13} , it is positively correlated with about half of the Y parameters. Four correlations are very weak, six relatively weak, and six are relatively strong. (Three are considered to be very strong; i.e., $|r| \ge 0.75$.) Equations for the relatively strong correlations are

$$\overline{R}_{MAX} = 16.048 + 14.321 \frac{\Delta}{GPV} \overline{R}_{13}$$
, (36)
 $(r = 0.88, S_{yx} = 18.63)$

$$\overline{R}_{CHM} = 9.337 + 7.349 \frac{\Delta}{GPV} \overline{R}_{13}$$
, (37)
(r = 0.89, S_{yx} = 9.40)

$$D_{CHM} = 73.010 - 2.422 \frac{\Delta}{GPV} \overline{R}_{13}$$
 , (38)
 $(r = -0.59, S_{yx} = 5.97)$

$$\overline{R}_{13}(t_{GPV}) = 4.647 + 9.844 \frac{\Delta}{GPV} \overline{R}_{13}$$
, (39)
 $(r = 0.75, S_{vx} = 21.37)$

$$\frac{\Delta}{\text{GPV}} \overline{R}_{13} = -3.006 - 0.318 \frac{\Delta}{\text{GPV}} \overline{R}_{13} ,$$

$$(r = -0.79, S_{yx} = 0.60)$$
(40)

$$\overline{R}_{13}(t_{GNV}) = 26.576 + 7.331 \frac{\Delta}{GPV} \overline{R}_{13}$$
 (41)
 $(r = 0.71, S_{VX} = 17.91)$

Concerning t_{GPV} , that is the time at which $\frac{\Delta}{GPV} \overline{R}_{13}$ occurs, it is positively correlated with about half of the parameters. Only two correlations are relatively strong, two are weak and twelve are very weak. Equations for the relatively strong correlations are

$$ASC_{CHM} = 13.737 + 0.418 t_{GPV}$$
 (42)
(r = 0.69, $S_{VX} = 4.57$)

and

$$\overline{R}_{13}(t_{GPV}) = 22.103 + 1.708 t_{GPV}$$
 (43)
 $(r = 0.55, S_{vx} = 22.103)$

Concerning \overline{R}_{MAX} , it is positively correlated with about half of the parameters. Five correlations are very weak, four are relatively weak, and seven are relatively strong (of which four are very strong). Equations for the relatively strong correlations are

$$\overline{R}_{CHM} = 1.352 + 0.511 \overline{R}_{MAX}$$
, (44)
(r = 1.00, $S_{yx} = 1.42$)

$$D_{CHM} = 70.603 - 0.125 \overline{R}_{MAX}$$
, (45)
(r = -0.50, $S_{yx} = 8.71$)

ASC =
$$59.071 - 0.094 \overline{R}_{MAX}$$
, (46)
(r = -0.56 , $S_{yx} = 5.62$)

$$_{\text{GPV}}^{\Delta} \overline{R}_{13} = 0.633 + 0.055 \overline{R}_{\text{MAX}}$$
, (47)
(r = 0.89, $S_{\text{VX}} = 1.15$)

$$\overline{R}_{13}(t_{GPV}) = 0.170 + 0.632 \overline{R}_{MAX}$$
, (48)
(r = 0.78, $S_{yx} = 20.31$)

$$\frac{\Delta}{\text{GNV}} \overline{R}_{13} = -3.255 - 0.017 \overline{R}_{\text{MAX}}$$
, (49)
(r = -0.69, S_{VX} = 0.72)

and

$$\overline{R}_{13}(t_{GNV}) = 12.198 + 0.565 \overline{R}_{MAX}$$
 (50)
 $(r = 0.88, S_{yx} = 11.84)$

Concerning ASC, it is positively correlated with only about 40 percent of the parameters. Five are very weak correlations, five are relatively weak, and six are relatively strong (of which two are very strong). Equations for the relatively strong correlations are

$$\overline{R}_{MAX} = 274.016 - 3.278 \text{ ASC}$$
 , (51)
 $(r = -0.56, S_{yx} = 33.15)$

$$\overline{R}_{CHM} = 141.082 - 1.669 \text{ ASC}$$
 , (52)
 $(r = -0.55, S_{yX} = 17.01)$

$$\Phi_{\text{MAX}} = 0.078 + 0.006 \text{ ASC}$$
 , (53)
 $(r = 0.83, S_{yx} = 0.03)$

$$\overline{R}_{13} (t_{GPV}) = 251.683 - 3.701 \text{ ASC}$$
 , (54)
 $(r = -0.77, S_{VX} = 20.51)$

$$t_{GNV} = 40.964 + 0.667 \text{ ASC}$$
 , (55)
 $(r = 0.53, S_{yx} = 7.23)$

$$\overline{R}_{13}(t_{GNV}) = 195.470 - 2.443 \text{ ASC}$$
 (56)
 $(r = -0.65, S_{vx} = 19.35)$

Concerning \overline{R}_{CHM} , it is positively correlated with about half of the parameters. Six are very weak correlations, three are relatively weak, and seven are relatively strong (of which four are very strong). Equations for the relatively strong correlations are

$$\overline{R}_{MAX} = -2.103 + 1.948 \overline{R}_{CHM}$$
 , (57)
(r = 1.00, $S_{VX} = 2.52$)

$$D_{CHM} = 71.017 - 0.246 \overline{R}_{CHM}$$
 , (58)
 $(r = -0.50, S_{VX} = 8.69)$

ASC =
$$59.262 - 0.183 \overline{R}_{CHM}$$
 , (59)
(r = -0.55 , $S_{VX} = 5.63$)

$$_{\text{GPV}}^{\Delta} \overline{R}_{13} = 0.488 + 0.107 \overline{R}_{\text{CHM}}$$
, (60)
(r = 0.89, $S_{\text{vx}} = 1.14$)

$$\overline{R}_{13}(t_{GPV}) = -1.145 + 1.229 \overline{R}_{CHM}$$
 , (61)
 $(r = 0.78, S_{yx} = 20.39)$

$$_{\text{GNV}}^{\Delta} \overline{R}_{13} = 7.294 - 0.034 \overline{R}_{\text{CHM}}$$
, (62)
 $(r = -0.71, S_{yx} = 0.70)$

$$\overline{R}_{13}(t_{GNV}) = 9.469 + 1.126 \overline{R}_{CHM}$$
 (63)
 $(r = 0.90, S_{VX} = 10.93)$

Finally, concerning ASC_{CHM}, about 70 percent are positively correlated. Seven are very weak correlations, seven are relatively weak, and only two are relatively strong correlations. Equations for the relatively strong correlations are

$$D_{CHM} = 82.495 - 1.004 \text{ ASC}_{CHM}$$
 (64)
 $(r = -0.63, S_{VX} = 7.78)$

and

$$t_{GPV} = 0.008 + 1.143 \text{ ASC}_{CHM}$$
 (65)
 $(r = 0.69, S_{VX} = 7.55)$

A second correlative listing, constructed similarly to Table 10 and really just a continuation of it, appears in Table 11. Here the X parameters are now sums of R_Z over certain time periods from \overline{R}_{MIN} occurrence. The three shown have been arbitrarily selected. In conjunction with \overline{R}_{MIN} , ASC_{CHM}, and $\frac{\Delta}{GPV}$ \overline{R}_{13} , they will allow parameters to be estimated about once every six months (from \overline{R}_{MIN} occurrence); thus, together they yield about five or so separate approximations to the parameters, some one to two years prior to \overline{R}_{MAX} occurrence.

Concerning $\sum_{t=0}^{12} R_Z(t)$, about two-thirds are positively correlated. Eight are very weak correlations, six are relatively weak, and two are relatively strong. Equations for the relatively strong correlations are

$$_{\text{GNV}}^{\Delta} \overline{R}_{13} = -3.892 - 0.012 \sum_{t=0}^{12} R_{Z}(t)$$
 (66)

TABLE 11. LINEAR REGRESSION COEFFICIENTS FOR CYCLE PARAMETERS BASED ON KNOWN SUMS OF MONTHLY MEAN SUNSPOT NUMBER FOR THE TIME INTERVALS 12, 18 AND 24 MONTHS

×	*	Ř _{MAX}	R _{CHM}	ASC _{CHM}	D _{CHM}	ASC	DES		MAX-MAX PERIOD	Фмах	Фмім	^t GPV	∆ GPV ^R 13	Ā ₁₃ (t _{GPV} I	^t GNV	∆ GNV ^R 13	R 13 ^{(t} GNV	Ř _{MIN} (SCN+1)
	,	0.36	0.42	0.20	-0.18	-0.22	0.34	0.21	0.32	-0.33	0.16	-0.16	0.44	0.18	-0.30	-0.52	0.54	0.21
12	S _{vx}	37.17	18.53	6.19	9.87	6.59	10.20	10.58	12.84	0.05	0.04	10.31	2.21	31.78	8.12	0.85	21.50	3.49
$\sum_{\mathbf{R}_{\mathbf{z}}(\mathbf{t})}$	a _{VX}	78.591	38.300	22.964	60.765	51,942	73.756	125.82	120.281	0.4116	0.6107	34.542	4.149	57.976	79.884	-3.892	42.259	3.664
· ·	byx	0.337	0.201	0,030	-0.042	-0.034	0.087	0.052	0.103	-0.0004	0.0002	-0.040	0.025	0.139	-0.061	-0.012	0.319	0.017
	,	0.55	0.58	-0.17	-0.17	-0.40	0.36	0.12	0.44	-0,46	0.02	-0.46	0.53	0.21	-0.42	-0.44	0.61	0.21
18	Syx	33.29	16.61	6.22	9.88	6.20	10.10	10.74	12.18	0.05	0.04	9.29	2.10	31.60	7.74	0.89	20.07	3.48
$\sum_{\mathbf{R}_{\mathbf{Z}}(\mathbf{t})}$	вух	47.376	23.451	29.698	61.553	56.49	70.960	127.711	113.005	0.4452	0.6330	45.196	2.921	52.109	84.287	-3.927	28.583	3.214
	byx	0.264	0.143	-0.013	-0.021	-0.032	0.048	0.015	0.072	-0.0003	0.0000	0.058	0.016	0.082	-0.043	-0.005	0.189	0.009
	•	0.77	0.78	-0.24	-0.24	-0.45	0,18	-0.10	0.35	-0.39	-0.21	-0.39	0.72	0.38	-0.42	-0.50	0.67	0.40
24	Syx	25.46	12.88	6.13	9.75	6.04	10.68	10.75	12,68	0.05	0.04	9.63	1.71	29.91	7.72	0.86	18.95	3.27
$\sum_{\mathbf{R}_{\mathbf{Z}}(t)}$	*yx	24.023	12.956	30.805	63.385	57.141	77.841	134,992	117,160	0.4232	0.6330	42,442	1.666	36.393	83.757	-3.544	26.704	1.065
	byx	0.164	0.095	-0.006	-0.013	-0.016	0.010	-0.006	0.026	-0.0001	0.0000	-0.022	0.009	0.066	-0.019	-0.003	0.091	0.008

$$(r = -0.52, S_{VX} = 0.85)$$

$$\overline{R}_{13}(t_{GNV}) = 42.259 + 0.319 \sum_{t=0}^{12} R_Z(t)$$
 (67)
(r = 0.54, S_{VX} = 21.50)

Concerning $\sum_{t=0}^{18} R_Z(t)$, about 60 percent are positively correlated. Six are very weak correlations, seven are relatively weak, and four are relatively strong. Equations for the relatively strong correlations are

$$\overline{R}_{MAX} = 47.376 + 0.264 \sum_{t=0}^{18} R_{Z}(t)$$
, (68)
(r = 0.55, $S_{VX} = 33.29$)

$$\overline{R}_{\text{CHM}} = 23.451 + 0.143 \sum_{t=0}^{18} R_{Z}(t)$$
 (69)

$$(r = 0.58, S_{yx} = 16.61)$$

$$_{\text{GPV}}^{\Delta} \overline{R}_{13} = 2.921 + 0.016 \sum_{t=0}^{18} R_{Z}(t) , \qquad (70)$$

$$(r = 0.53, S_{yX} = 2.10)$$

and

$$\overline{R}_{13}(t_{GPV}) = 28.583 + 0.189 \sum_{t=0}^{18} R_Z(t)$$
 (71)

$$(r = 0.61, S_{yx} = 20.07)$$

Finally, concerning $\sum_{t=0}^{24} R_Z(t)$, about half are positively correlated. Five are very weak correlations, seven are relatively weak, and five are relatively strong correlations. Equations for the relatively strong correlations are

$$\overline{R}_{MAX} = 24.023 + 0.164 \sum_{t=0}^{24} R_Z(t)$$
 , (72)

$$(r = 0.77, S_{yX} = 25.46)$$

$$\overline{R}_{CHM} = 12.956 + 0.085 \sum_{t=0}^{24} R_{Z}(t)$$
 , (73)

$$(r = 0.78, S_{VX} = 12.88)$$

$$\frac{\Delta}{\text{GPV}} \overline{R}_{13} = 1.666 + 0.009 \sum_{t=0}^{24} R_{Z}(t) ,$$
(74)

$$(r = 0.72, S_{VX} = 1.71)$$

$$(r = -0.50, S_{VX} = 0.86)$$

$$\overline{R}_{13}(t_{GNV}) = 26.704 + 0.091 \sum_{t=0}^{24} R_Z(t)$$
 (76)

$$(r = 0.67, S_{VX} = 18.95)$$

Tables 10 and 11 allow parametric approximations to be made, based on selected early cycle parameters. In addition to these early cycle parameters, there are three late cycle parameters that may be of interest for estimating DES, $\overline{R}_{MIN}(SCN+1)$ and MIN-MIN PERIOD. These include $\frac{\Delta}{GNV}$ \overline{R}_{13} , t_{GNV} , and t_{GNV} , and t_{GNV} , and t_{GNV} , and t_{GNV} , are cycle parametric correlations.

TABLE 12. LINEAR REGRESSION COEFFICIENTS FOR SELECTED CYCLE PARAMETERS BASED ON KNOWN SELECTED LATE OCCURRING CYCLE PARAMETERS

	STATISTICAL		Y PARAME	TER
X PARAMETER	PARAMETERS	DES	R _{MIN} (SCN + 1)	MIN-MIN PERIOD
$GNV^{igstyle R}$ 13	r	0.22	-0.25	0.45
	s_{yx}	10.58	3.45	9.65
	a_{yx}	96.080	0.879	157.482
	b _{yx}	2.413	-0.895	4.945
^t GNV	r	-0.38	-0.25	-0.05
	s_{yx}	10.04	3.45	10.80
	^a yx	118.831	13.160	136.370
	b_{yx}	-0.484	-0.104	-0.065
D _{CHM}	r	-0.13	0.29	0.001
	s_{yx}	10.75	3.41	10.82
	a _{yx}	91.479	-0.160	131.564
	$\mathbf{b_{yx}}$	-0.143	0.102	0.001

Inspection of Table 12 reveals that none of these late cycle parameters are strongly correlated with DES, $\overline{R}_{MIN}(SCN+1)$ or MIN-MIN PERIOD. The strongest correlation appears to be between $\frac{\Delta}{GNV}$ \overline{R}_{13} and MIN-MIN PERIOD which has r=0.45 and $S_{yx}=9.65$. The regression equation is

MIN-MIN PERIOD = 157.482 + 4.945
$$\frac{\Delta}{GNV} \overline{R}_{13}$$
 (77)

(It should be noted that equation (77) has the highest correlation for MIN-MIN PERIOD of the cycle parameters tested.)

One additional parameter of interest remains to be discussed. This is MAX-MAX PERIOD. A comparison of MAX-MAX PERIOD with DES and MIN-MIN PERIOD appears below:

MAX-MAX PERIOD =
$$40.047 + 1.099$$
 DES
$$(r = 0.88, S_{yx} = 5.17)$$
(78)

MAX-MAX PERIOD =
$$26.737 + 0.798$$
 (MIN-MIN PERIOD) . (79)
(r = 0.64 , $S_{yx} = 8.35$)

Both correlations are relatively strong and positive. Equation (78) allows one to estimate ASC(SCN + 1), since

$$MAX-MAX PERIOD = DES(SCN) + ASC(SCN + 1) (80)$$

IV. DISCUSSION

A. General Remarks Concerning Sunspot Cycles

Of all the aforementioned parameters, five are probably of the most interest for any given cycle. These include: \overline{R}_{MIN} occurrence date, \overline{R}_{MIN} value, \overline{R}_{MAX} occurrence date, \overline{R}_{MAX} value, and cycle duration (or MIN-MIN PERIOD). From these, we can crudely determine \overline{R}_{13} values as a function of time and determine many of the other cycle parameters. In Figure 19, it was observed that the trend for both \overline{R}_{MIN} and \overline{R}_{MAX} values, as a function of SCN, is downward for cycles 8 through 14 and upward for cycles 14 through 20. The correlation coefficients are about $|r| \geq 0.7$, suggesting that \overline{R}_{MIN} and \overline{R}_{MAX} may be related to each other, in a positive sense. Indeed, this is found to be true, although the correlation is not as strong as might be desired. If SCN 19 is excluded, the correlation is much stronger; if SCN 19 is excluded and sunspot cycles are considered in terms of long-period and short-period cycle groups, then the correlation is very much higher. Thus, high-valued \overline{R}_{MIN} cycles tend to have high- \overline{R}_{MAX} values, rise to maximum more quickly (i.e., they have short ASC), and decay more slowly (i.e., they have long DES) than low-valued \overline{R}_{MIN} cycles, and these parameters appear to be distributed by cycle duration. The correlation between DES and ASC is also very high ($|r| \geq 0.8$) when the long- and short-period cycle grouping is employed, as it is for \overline{R}_{MAX} and ASC.

B. SCN 21 Parametric Estimations

The preceding sections have identified mean values, standard deviations, ranges and correlations between easily observable parameters and SCN, based on cycles 8 through 20. This paper will now examine what these results have to say about cycle 21 – the present sunspot cycle which is now about 4 years past solar maximum and about 7 years into the cycle. To do this, the beginning point must be selected parameters for the last observed cycle – SCN 20 – which began in October 1964 (\overline{R}_{MIN} occurrence) and peaked in November 1968 (\overline{R}_{MAX} occurrence). (See Table 2.)

The first problem to be addressed is the determination of $\overline{R}_{MIN}(SCN\ 20\ +\ 1)$ occurrence, marking the end of SCN 20 and the beginning of SCN 21. (Please note that $\overline{R}_{MIN}(SCN\ 21)$ has already occurred

and, therefore, is known; this is merely an exercise to determine the ability to approximate such things as \overline{R}_{MIN} occurrence date for a future cycle based on the findings from the previous sections.) Since cycle 20 began in October 1964, it is known, based on mean values, that its MIN-MIN PERIOD approximately equals 132 \pm 10 months (i.e., 1-sigma accuracy, corresponding to about 68 percent chance of occurrence within the stated range); consequently, \overline{R}_{MIN} (SCN 21) occurrence date is approximated to be October 1975, give or take about 10 months. This same date can be deduced by applying the mean value of DES to the \overline{R}_{MAX} (SCN 20) occurrence date.

Another projection comes from use of equation (77) in conjunction with $\frac{\Delta}{GNV}$ \overline{R}_{13} (SCN 20). Recall,

MIN-MIN PERIOD = 157.482 + 4.945
$$\frac{\Delta}{GNV} \overline{R}_{13}$$
 (77)

Since $_{GNV}^{\Delta}$ \overline{R}_{13} (SCN 20) = -5.3, MIN-MIN PERIOD equals 131.27 and \overline{R}_{MIN} (SCN 21) occurrence date is then October 1964 + 131.27 months or about September 1975 (±10 months). Several other \overline{R}_{MIN} (SCN 21) occurrence dates can be projected based on various regression equations summarized in Table 10, each with a 1-sigma confidence limit of about 10 months. These include April 1976 (based on DES versus \overline{R}_{MIN}), February 1976 (based on MIN-MIN PERIOD versus \overline{R}_{MIN}), October 1975 (based on MIN-MIN PERIOD versus ASC_{CHM}) and August 1975 (based on MIN-MIN PERIOD versus ASC_{CHM}) and August 1975 (based on MIN-MIN PERIOD versus ASC_{CHM}). Still another date, March 1976 ± 18 months, comes from equations (22) and (23) and Table 9.

A different strategy might be to determine MIN-MIN PERIOD based on a comparison of SCN 20 with other "similar" cycles. SCN 20 could be categorized as a low-valued \overline{R}_{MAX} cycle (i.e., $\overline{R}_{MAX} < 116.2$; Table 4), based on its \overline{R}_{MAX} value at cycle maximum. Thus, its MIN-MIN PERIOD would have been expected to be about 133 months ± 8 months, suggesting an \overline{R}_{MIN} (SCN 21) occurrence date of November 1975. Inspection of Table 4 reveals, however, that (excluding SCN 20), four of six low-valued \overline{R}_{MAX} cycles were also long-period cycles (including cycle 20 makes it 5 of 7), having a MIN-MIN PERIOD of about 138 months ± 4 months; so, one might expect SCN 20 to have an \overline{R}_{MIN} (SCN 20 + 1) occurrence date of April 1976. As previously noted, MIN-MIN PERIOD is not evenly distributed. Instead, there appears to be a long-period cycle group and a short-period cycle group. If this logic is applied to SCN 20, then the \overline{R}_{MIN} (SCN 21) occurrence date would be either December 1974 ± 3 months, if SCN 20 is a short-period cycle (see Table 4), or it would be June 1976 ± 5 months, if SCN 20 is a long-period cycle. [These same dates can be obtained from equation (24).] Thus, if the projection is based upon \overline{R}_{13} values post-June 1975, one would have to conclude that \overline{R}_{MIN} (SCN 21) occurrence date would be June 1976.

A final point is that $\overline{R}_{MIN}(SCN+1)$ is approximately 5.6 ± 3.3, yielding the 3-sigma upper limit of $\overline{R}_{MIN}(SCN+1)$ to be about 15.5. Thus, if \overline{R}_{13} is greater than 15.5 in December 1974, then the occurrence date for $\overline{R}_{MIN}(SCN-21)$ is very highly probable to occur at the later projected date. In December 1974, cycle 20 had $\overline{R}_{13} \sim$ 25, almost 6 sigma too high to be considered \overline{R}_{MIN} ; so, it could have been determined as early as December 1974 that minimum for cycle 21 would not occur until about June 1976 (which it did).

The second problem to be addressed is the determination of the value of \overline{R}_{MIN} at \overline{R}_{MIN} (SCN 21) occurrence. (Again, this parameter is already known.) Based on mean value and 1-sigma confidence, \overline{R}_{MIN} (SCN 21) would be expected to be about 5.2 ± 2.7 (Table 2). Low- \overline{R}_{MAX} cycles have \overline{R}_{MIN} (SCN + 1) = 5.1 ± 3.2 and long-period cycles have \overline{R}_{MIN} (SCN + 1) = 4.6 ± 3.4 (Table 4). Figures 9A, 10A and 19 reveal that the trend of the last 8 sunspot cycles has been toward higher \overline{R}_{MIN} values. Equation (19) suggests a value of \overline{R}_{MIN} (SCN 21) = 8.7 ± 2.1, if the trend continues upward. Based on parametric correlation, \overline{R}_{MIN} (SCN 21) = 5.3 ± 3.2 [Table 10, \overline{R}_{MIN} (SCN + 1) versus \overline{R}_{MAX}] or 6.2 ± 3.3 [Table 24]

11, $\overline{R}_{MIN}(SCN + 1)$ versus $\sum_{t=0}^{24} R_Z(t)$; Appendix F lists values of $\sum R_Z(t)$ for 12-, 18-, and 24-month

periods for SCN 8 through 21]. Thus, a value between 1.2 and 10.8 would be expected for $\overline{R}_{MIN}SCN$ 21). The best guess is probably 8.7 ± 2.1, shown above, since its correlation is relatively strong (r = 0.7). $\overline{R}_{MIN}(SCN 21)$ was observed to be 12.2; i.e., about 1.7 S_{yx} units greater in value than its predicted value or about 2.6 s units greater than the mean value, based on cycles 8 through 20. Therefore, the upward trend in \overline{R}_{MIN} continued, at least through cycle 21.

Now, given \overline{R}_{MIN} and its occurrence date, we can estimate the remaining cycle parameters on the basis of the relationships identified in Table 10. As the cycle progresses, we can update these estimates based on the occurrence of other early cycle parameters; namely, ${}^{\Delta}_{GPV}$ \overline{R}_{13} , t_{GPV} , \overline{R}_{MAX} , ASC, \overline{R}_{CHM} , and ASC_{CHM}.

Table 13 compares observed values (for SCN 21) of the aforementioned cycle parameters with their respective mean values (i.e., the means of cycles 8 through 20, LOW- \bar{R}_{MAX} cycles, HIGH- \bar{R}_{MAX} cycles , LONG-PERIOD cycles and SHORT-PERIOD cycles; Table 4) and those based on \bar{R}_{MIN} and $\frac{\Delta}{GPV}$ \bar{R}_{13} -correlation equations. (Please note that these two early cycle parameters usually have the highest Pearson correlation coefficient; exceptions are noted as footnotes in Table 13.) The numbers in parentheses are the standard deviations in the case of mean value numbers or the standard error of estimates for the correlation-equation derived values.

As an example, it is seen that, based on the mean of cycles 8 through 20 and 1-sigma confidence, $\overline{R}_{MAX}(SCN\ 21)=116.2\pm36.7$. If cycle 21 is a LOW- \overline{R}_{MAX} cycle, then $\overline{R}_{MAX}(SCN\ 21)=88.4\pm15.8$; however, if cycle 21 is a HIGH- \overline{R}_{MAX} cycle, then $\overline{R}_{MAX}(SCN\ 21)=148.6\pm25.8$. If cycle 21 is a LONG-PERIOD cycle, $\overline{R}_{MAX}(SCN\ 21)=101.1\pm26.3$; but, if cycle 21 is a SHORT-PERIOD cycle, then $\overline{R}_{MAX}(SCN\ 21)=133.8\pm39.1$. Based on $\overline{R}_{MIN}(SCN\ 21)=12.2$, $\overline{R}_{MAX}(SCN\ 21)$ is computed to be 142.7 \pm 38.2; based on $\frac{\Delta}{GPV}$ $\overline{R}_{13}(SCN\ 21)=+8.1$, $\overline{R}_{MAX}(SCN\ 21)$ is 132.0 \pm 18.5. So, both $\overline{R}_{MIN}(SCN\ 21)$ and $\frac{\Delta}{GPV}\overline{R}_{13}(SCN\ 21)$ suggest $\overline{R}_{MAX}(SCN\ 21) \geqslant \overline{R}_{MAX}(MEAN\ Cycles\ 8$ through 20), implying that cycle 21 will probably be a HIGH- \overline{R}_{MAX} cycle and also of short cycle-duration (i.e., a SHORT-PERIOD cycle); it follows, then, that the best guess estimate for $\overline{R}_{MAX}(SCN\ 21)$ is 141.2 \pm 32.5 (an average of the two mean cycle numbers; i.e., HIGH- \overline{R}_{MAX} plus SHORT-PERIOD cycle estimates divided by 2). [Note: $\overline{R}_{MAX}(SCN\ 21)$ would have been estimated to be 176.5 \pm 34.2 based on equation (20) and assuming that the \overline{R}_{MAX} versus SCN upward trend continued.] $\overline{R}_{MAX}(SCN\ 21)$ is observed to be 164.5 or about 0.6 s unit above the mean based on HIGH- \overline{R}_{MAX} cycles; it is about

Table 13. Comparison of observed values of selected cycle parameters for cycle 21 with various estimates based on known \bar{R}_{MIN}^- and $_{GPV}^\Delta$ \bar{R}_{13}^- based linear regression

EQUATIONS AND ON MEAN VALUES FOR HIGH- AND LOW- $\overline{R}_{\mbox{\scriptsize MAX}}$ AND LONG- AND SHORT-PERIOD CYCLE GROUPINGS

PARAMETER	OBSERVED VALUE	CYCLE 8-20 MEAN VALUE	R _{MIN} -BASED ESTIMATED VALUE	Δ GPVR ₁₃ BASED ESTIMATED VALUE	LOW-R _{MAX} MEAN VALUE	HIGH-R _{MAX} MEAN VALUE	LONG PERIOD MEAN VALUE	SHORT PERIOD
R _{MAX}	164.5	116.2 (36.7)	142.7 (38.2)	132.0 (18.5)	88.4 (15.8)	148.6 (25.8)	101.1 (26.3)	133.8 (39.1)
1 R _{CHM}	88.4	60.7 (18.8)	77.8 (19.1)	68.9 (9.4)	46.4 (8.5)	77.5 (12.6)	53.3 (14.3)	69.4 (19.7)
2 ASC _{CHM}	24	26.3 (5.8) 24.8 (2.9)*	32.7 (5.7)	27.0 (6.1)	24.0 (2.8)	29.0 (7.1) 26.0 (2.5)*	27.0 (7.6) 24.2 (3.2)*	25.5 (2.3)
3 DCHM	_	56.1 (9.2) 57.5 (8.1)*	51.3 (9.8)	53.4 (6.0)	61.3 (8.3)	50.0 (5.9) 52.2 (3.7)*	57.9 (12.0) 61.0 (9.9)*	54.0 (3.2)
4 ASC	42	48.2 (6.2)	46.0 (6.7)	46.7 (5.9)	51.6 (4.5)	44.2 (5.5)	50.1 (5.6)	45.8 (6.1)
DES	_	83.5 (10.0	92.9 (10.1)	83.2 (10.8)	81.6 (10.7)	85.7 (8.5)	89.9 (7.9)	76.0 (6.3)
MIN-MIN PERIOD	_	131.6 (10.0)	138.9 (10.4)	129.9 (10.1)	133.1 (8.2)	129.8 (11.4)	140.0 (4.7)	121.8 (3.3)
MAX-MAX PERIOD	_	131.8 (12.5)	140.4 (13.1)	133.1 (13.2)	128.4 (11.1)	135.7 (12.8)	138.1 (12.1)	124.3 (8.0)
Φ_{MAX}	_	0.37 (0.05)	0.33 (0.05)	0.36 (0.05)	0.39 (0.05)	0.34 (0.03)	0.36 (0.04)	0.38 (0.05)
Φ_{MIN}	_	0.63 (0.04)	0.66 (0.04)	0.63 (0.04)	0.63 (0.04)	0.63 (0.03)	0.65 (0.02)	0.61 (0.04)
tGPV	24	30.1 (9.6) 28.8 (8.9)	31.4 (10.4)	31.1 (10.2)	27.3 (10.5)	33.3 (7.2) 31.0 (5.5)*	28.6 (10.8) 25.8 (9.1)*	31.8 (7.7)
GPVR13	+8.1	+7.0 (2.3)	+8.8 (2.3)	x	+5.4 (1.6)	+8.9 (1.3)	+5.8 (2.1)	+8.3 (1.6)
R ₁₃ (t _{GPV})	89.3	73.5 (29.7)	90.6 (32.3)	84.4 (21.37)	50.6 (18.8)	100.2 (13.6)	57.1 (21.8)	92.6 (26.1)
^t GNV	_	73.1 (7.8)	65.9 (8.0)	72.2 (8.3)	75.3 (8.9)	70.5 (5.3)	73.0 (8.4)	73.2 (7.2)
gNVR ₁₃	_	-5.2 (0.9)	-6.4 (0.9)	-5.6 (0.6)	-4.7 (0.7)	-5.9 (0.7)	-4.7 (0.7)	-5.8 (0.7)
R 13 (tGNV)	_	77.8 (23.4)	110.3 (21.4)	86.0 (17.9)	60.2 (17.0)	98.4 (8.1)	74.7 (20.9)	81.5 (25.5)
RMIN(SCN +1)	_	5.6 (3.3)	7.2 (3.5)	6.1 (3.4)	5.1 (3.2)	6.1 (3.3)	4.6 (3.4)	6.7 (2.8)

*EXCLUDES SCN 9

¹ \overline{R}_{CHM} = 1.352 + 0.511 \overline{R}_{MAX} ; THEREFORE, \overline{R}_{CHM} (SCN 21) = 85.4 (r = 1.00, S_{VX} = 1.4)

² ASC_{CHM} = 13.737 + 0.418 t_{GPV} ; THEREFORE ASC_{CHM} (SCN 21) = 23.8, IN VERY GOOD AGREEMENT WITH THE OBSERVED VALUE (r = 0.69, S_{YX} = 4.6)

³ D_{CHM} = 82.495 - 1.004 ASC_{CHM}; THEREFORE D_{CHM} (SCN 21) = 58.4 (r = -0.63, Syx = 7.8)

⁴ ASC = 59.071 - 0.094 \overline{R}_{MAX} ; THEREFORE ASC (SCN 21) = 43.6 (r = -0.56, S $_{VX}$ = 5.6)

0.8 s unit above the mean based on SHORT-PERIOD cycles, about 0.6 S_{yx} unit above the predicted value based on \overline{R}_{MIN} correlation, and about 1.8 S_{yx} units above the predicted value based on $\frac{\Delta}{GPV}\overline{R}_{13}$ correlation. It is only 0.4 S_{vx} unit below the value predicted from equation (20).

As a second example, based on the mean of cycles 8 through 20, $ASC_{CHM}(SCN\ 21) = 26.3\pm 5.8$; excluding SCN 9 yields $ASC_{CHM}(SCN\ 21) = 24.8\pm 2.9$. Since, from above, cycle 21 was expected to be a HIGH- \bar{R}_{MAX} cycle, excluding SCN 9, $ASC_{CHM}(SCN\ 21) = 26.0\pm 2.5$ would be estimated. [The SHORT-PERIOD cycle mean suggests $ASC_{CHM}(SCN\ 21) = 25.5\pm 2.3$.] Based on $\bar{R}_{MIN}(SCN\ 21)$ and $\frac{\Delta}{GPV}$ $\bar{R}_{13}(SCN\ 21)$, $AS_{CHM}(SCN\ 21) = 32.7\pm 5.7$ and 27.0 ± 6.1 , are computed, respectively. Based on $t_{GPV}(SCN\ 21)$, $ASC_{CHM}(SCN\ 21) = 23.8\pm 4.6$ is computed (footnote 2, Table 13). The best guess estimate for $ASC_{CHM}(SCN\ 21)$ is probably 25.8 ± 2.4 , based on the average of HIGH- \bar{R}_{MAX} and SHORT-PERIOD cycle numbers. $ASC_{CHM}(SCN\ 21)$ is observed to be 24 months or about 0.8 s unit below the mean of HIGH- \bar{R}_{MAX} cycles, about 0.7 s unit below the mean of SHORT-PERIOD cycles, about 0.3 s unit below the mean of cycles 8 through 20 (excluding SCN\ 9), about 1.5 S_{yx} units below the \bar{R}_{MIN} -correlated predicted value, and about 0.5 S_{yx} unit below the $\frac{\Delta}{GPV}$ \bar{R}_{13} -correlated predicted value.

Having a good guess for ASC_{CHM} (i.e., 25.8 \pm 2.4), the estimate of \overline{R}_{MAX} (SCN 21) can be checked by means of the definition of \overline{R}_{CHM} ; recall that \overline{R}_{CHM} is defined in equation (3) to be

$$\overline{R}_{CHM} = \frac{\overline{R}_{MIN} + \overline{R}_{MAX}}{2}$$
 (3)

occurring at ASC_{CHM}. So,

$$\overline{R}_{MAX}(SCN\ 21) = 2\ \overline{R}_{CHM}(SCN\ 21) - \overline{R}_{MIN}(SCN\ 21) \quad . \tag{81}$$

 $\overline{R}_{MIN}(SCN~21)$ equals 12.2 and $\overline{R}_{CHM}(SCN~21)$ can be estimated by averaging the \overline{R}_{13} values between 23 months after \overline{R}_{MIN} occurrence (i.e., t = 23) to about 28 months after \overline{R}_{MIN} occurrence (i.e., t = 28), based on a 1-sigma spread; thus, \overline{R}_{CHM} would be estimated to be 98.9 ± 10.0 and $\overline{R}_{MAX}(SCN~21)$ = 185.6 ± 10.0. Based on this analysis, $\overline{R}_{MAX}(SCN~21)$ is about 2.1 s units below the predicted value. Instead of using the above average for \overline{R}_{CHM} , if the ASC_{CHM} = 24.8 ± 2.9 had been used based on the mean of cycles 8 through 20 (excluding SCN 9), $\overline{R}_{CHM}(SCN~21)$ can be approximated to be 95.8 ± 12.1 and $\overline{R}_{MAX}(SCN~21)$ = 179.4 ± 12.1, or the observed $\overline{R}_{MAX}(SCN~21)$ is about 1.2 s units below the predicted value. [It is very interesting to note that for cycles 19, 20 and 21, ASC_{CHM} has been 23, 23 and 24, respectively, and that t_{GPV} occurred almost simultaneously with it; t_{GPV} was 22, 21 and 24 for cycles 19, 20 and 21, respectively. Thus, monitoring $\frac{\Delta}{GPV}$ \overline{R}_{13} and assuming that ASC_{CHM} and t_{GPV}

occur at the same time suggests that $\overline{R}_{CHM} = \overline{R}_{13}(t_{GPV})$ which implies that \overline{R}_{MAX} equals 193.6, 91.0 and 166.4 for cycles 19, 20 and 21, respectively, in very close agreement to observed values (Table 2).]

 D_{CHM} , the time interval in months when $\overline{R}_{13} \ge \overline{R}_{CHM}$, is equal to about 57.5 \pm 8.1 months, based on the mean cycles of 8 through 20 (excluding SCN 9). Based on cycle 21 being a HIGH- \overline{R}_{MAX} cycle, $D_{CHM} = 52.2 \pm 3.7$ (again, excluding 9). Based on cycle 21 being a SHORT-PERIOD cycle, $D_{CHM} = 54.0 \pm 3.2$. \overline{R}_{MIN}^- and \overline{G}_{PV}^{A} \overline{R}_{13}^- -correlation equations predict $D_{CHM}(SCN\ 21) = 51.3 \pm 9.8$ and 53.4 \pm 6.0 months, respectively. The best guess estimate, based on the average of mean values for HIGH- \overline{R}_{MAX} and SHORT-PERIOD cycles, is probably $D_{CHM} = 53.1 \pm 3.5$. Now, \overline{R}_{CHM} is computed to be 88.4, based on observed values for \overline{R}_{MIN} and \overline{R}_{MAX} equal to 12.2 and 164.5, respectively. \overline{R}_{13} exceeded \overline{R}_{CHM} about t = 24, corresponding to June 1978. Therefore, \overline{R}_{13} should be $\geq \overline{R}_{CHM}$ until about November 1982 \pm 3 or 4 months. Based on the prediction of ASC_{CHM} (i.e., $ASC_{CHM} = 25.8 \pm 2.4$), \overline{R}_{13} could have been predicted to exceed \overline{R}_{CHM} from about August 1978 \pm 2 or 3 months to about January 1983 \pm 3 or 4 months.

As another example, based on the mean of cycles 8 through 20, ASC(SCN 21) = 48.2 \pm 6.2 months. Assuming that cycle 21 is a HIGH- \overline{R}_{MAX} cycle, ASC(SCN 21) = 44.2 \pm 5.5; if cycle 21 is a SHORT-PERIOD cycle, ASC(SCN 21) = 45.0 \pm 6.1. Thus, the best guess estimate it about ASC(SCN 21) = 45.0 \pm 5.8. \overline{R}_{MIN} and $\frac{\Delta}{GPV}$ \overline{R}_{13} -correlation equations predict ASC(SCN 21) = 46.0 \pm 6.7 and 46.7 \pm 5.9 months, respectively. Based on the mean of MAX-MAX PERIOD and knowing that DES(SCN 20) = 91 months, ASC(SCN 21) = 40.8 \pm 12.5 months could have been predicted. Based on DES(SCN 20) = 91 and the MAX-MAX PERIOD-correlation equation [equations (78) and (80)], ASC(SCN 21) = 49.1 \pm 5.2 months could have been predicted. The actual value for ASC(SCN 21) is observed to be 42 or about 1.1 s units below that predicted for a HIGH- \overline{R}_{MAX} cycle, 0.4 s unit below that predicted for a SHORT-PERIOD cycle, 0.6 S_{yx} unit below that predicted from the \overline{R}_{MIN} -correlation equation, and 0.8 S_{yx} unit below that predicted from the \overline{R}_{MIN} -correlation equation, and 0.8 S_{yx} unit below that predicted from the \overline{R}_{MIN} -correlation equation.

Having the good guess for ASC (i.e., 45.0 ± 5.8), \overline{R}_{MAX} could have been predicted to occur about March 1980, give or take about 5 or 6 months. Based on the mean, \overline{R}_{MAX} (SCN 21) could have been projected to occur about June 1980 \pm 6 months. The actual date of \overline{R}_{MAX} (SCN 21) occurrence is December 1979.

Additional examples can be worked for each of the parameters; however, to avoid undue discussion, a simple summary is provided in Table 14. There, the observed parameters are again given, as are the mean of the parametric values predicted by the \overline{R}_{MIN} and \overline{GPV} \overline{R}_{13} -correlation equations and the mean of those values predicted by HIGH- \overline{R}_{MAX} and SHORT-PERIOD cycles. One observes that the two means are usually in very close agreement with each other and that often they are within one unit of standard deviation (or one unit of standard error of estimate) with the observed value for cycle 21. Assuming that cycle 21 is a HIGH- \overline{R}_{MAX} and SHORT-PERIOD cycle, cycle 21 will have DES = 80.9 \pm 7.4 months and MIN-MIN PERIOD = 42 + (80.9 \pm 7.4) = 122.9 \pm 7.4 months, indicating \overline{R}_{MIN} (SCN 22) to occur about September to December 1986 \pm 7 months with a value of about 6.4 \pm 3.1. (If equation (19b) is used and a continuing upward trend in \overline{R}_{MIN} versus SCN is assumed, then \overline{R}_{MIN} (SCN 22) = 9.6 \pm 2.1.)

TABLE 14. COMPARISON OF OBSERVED VALUES OF CYCLE PARAMETERS FOR CYCLE 21 WITH MEAN VALUES OF SELECTED ESTIMATES

PARAMETER	OBSERVED	MEAN 1*	MEAN 2*
R _{MIN}	12.2	_	5.6 (2.4)
\overline{R}_{MAX}	164.5	137.4 (28.4)	141.2 (32.5)
R _{CHM}	88.4	73.3 (14.2)	73.5 (16.3)
ASC _{CHM}	24	29.9 (5.9)	25.8 (2.4) ^b
D _{CHM}	_	52.3 (7.9)	53.1 (3.5) ^b
ASC	42	46.3(6.3)	45.0 (5.8)
DES	-	88.0 (10.5)	80.9 (7.4)
MIN-MIN PERIOD	_	134.4 (10.2)	125.8 (7.4)
MAX-MAX PERIOD	_	136.8 (13.2)	130.0 (10.4)
$^{\Phi}$ MAX	_	0.35 (0.05)	0.36 (0.04)
$^{\Phi}$ MIN	_	0.64 (0.04)	0.62 (0.04)
t _{GPV}	24	31.2 (10.3)	31.4 (6.6)
GPV ^R 13	+8.1	+8.8 (2.3) ^a	+8.6 (1.5)
R ₁₃ (t _{GPV})	89.3	87.5 (26.9)	96.4 (19.9)
^t GNV	~	69.0 (8.1)	71.9 (6.3)
GNV ^R 13	_	-6.0 (0.7)	-5.9 (0.7)
R ₁₃ (t _{GNV})	_	98.1 (19.2)	90.0 (16.8)
R _{MIN} (SCN + 1)	_	6.6 (3.4)	6.4 (3.1)
R _{MIN} OCCURRENCE DATE	JUN 1976	_	_
R _{MAX} OCCURRENCE DATE	DEC 1979	APR(±6) 1980	MAR(±6) 1980
R _{MIN} (SCN + 1) OCCURRENCE DATE	_	AUG (±10) 1987	DEC (±7) 1986
R _{CHM} OCCURRENCE DATES	JUN 1978 -	DEC (±6) 1978 APR (±9) 1983	AUG (±2) 1978 JAN (±4) 1983

^{*}MEAN 1 IS THE MEAN OF COMPUTED VALUES BASED ON \overline{R}_{MIN} AND $\overrightarrow{GPV}^{\overline{R}}$ 13 CORRELATION EQUATIONS.

^{**}MEAN 2 IS THE MEAN OF HIGH- $\overline{\mathtt{R}}_{\mathsf{MAX}}$ AND SHORT-PERIOD CYCLES.

a RMIN-CORRELATION EQUATION ONLY b EXCLUDES SCN 9

TABLE 15. COMPARISON OF OBSERVED VALUES OF CYCLE PARAMETERS FOR CYCLE 21 WITH ESTIMATES BASED ON SUMS OF MEAN MONTHLY SUNSPOT NUMBER (12, 18 AND 24 MONTHS) LINEAR REGRESSION EQUATIONS

PARAMETER	OBSERVED	$\Sigma_{\mathbf{R}_{\mathbf{Z}}}^{12}(\mathbf{t})$	18 Σ R _z (t) ο	$\Sigma R_z(t)$
R _{MAX}	164.5	147.1 (37.2)	156.9 (33.3)	174.0 (25.5)
R _{CHM}	88.4	79.2 (18.5)	82.8 (16.6)	90.7 (12.9)
ASC _{CHM}	24	29.1 (6.2)	24.3 (6.2)	23.5 (6.1)
D _{CHM}	_	52.2 (9.9)	52.8 (9.9)	51.5 (9.8)
ASC	42	45.0 (6.6)	43.2 (6.2)	42.5 (6.0)
DES	_	91.4 (10.2)	90.9 (10.1)	87.0 (10.7)
MIN-MIN PERIOD	_	136.4 (10.6)	133.9 (10.7)	129.5 (10.8)
MAX-MAX PERIOD	_	141.2 (12.8)	142.9 (12.2)	140.9 (12.7)
$^{\Phi}$ MAX	_	0.33 (0.05)	0.32 (0.05)	0.33 (0.05)
Φ MIN	_	0.65 (0.04)	0.63 (0.04)	0.63 (0.04)
^t GPV	24	26.4 (10.3)	21.1 (9.3)	22.3 (9.6)
GPV ^R 13	+8.1	+9.2(2.2)	+9.6 (2.1)	+9.9 (1.7)
R ₁₃ (t _{GPV})	89.3	86.2 (31.8)	86.1 (31.6)	96.7 (29.9)
^t GNV	-	67.5 (8.1)	66.4 (7.7)	66.4 (7.7)
GNV ^R 13	-	-6.3 (0.9)	-6.0 (0.9)	-6.3 (0.9)
R ₁₃ (t _{GNV})	_	107.1 (21.5)	107.0 (20.1)	109.9 (19.0)
R _{MIN} (SCN + 1)	-	7.1 (3.5)	6.9 (3.5)	8.4 (3.3)

As the cycle parameters were estimated from the linear regression equations of Table 10, so could they also be estimated from Table 11, using sums of R_Z over selected periods of time. Examination of Table 11 reveals that usually a higher Pearson correlation coefficient is associated with the sum of R_Z -correlation based on larger time periods, suggesting that, perhaps, $\sum_{t=0}^{24} R_Z(t)$ -correlation equations will yield good results for comparison to observed cycle 21 parameters. Table 15 compares observed cycle parameters for cycle 21 based on the correlation equations contained in Table 11. The rightmost column corresponds to those parametric values based on $\sum_{t=0}^{24} R_Z(t)$ -correlation equations. Thus, based on $\sum_{t=0}^{24} R_Z(t)$ -correlation equations. Thus, based on $\sum_{t=0}^{24} R_Z(t)$ -correlation equation projections appear to be in close approximation to already observed cycle parameters. On the basis of $\sum_{t=0}^{24} R_Z(t)$, cycle 21 will end about March or April 1987 ± 11 months with a value of $\overline{R}_{MIN}(SCN 22) = 8.4 \pm 3.3$.

In a previous section (Section III.D), several curves of $R_Z(t)$ and $\overline{R}_{13}(t)$ were drawn, representing the mean cycle curve (in terms of R_Z and \overline{R}_{13} and based on cycles 8 through 20), HIGH- and LOW- \overline{R}_{MAX} cycle curves, and LONG- and SHORT-PERIOD cycle curves. Each of these curves was compared with cycles 8 through 20 and these comparisons are given in Appendices B through E. In this section, these same "mean" curves are plotted as a function of t, time from \overline{R}_{MIN} occurrence. Figure 24 compares R_Z(SCN 21) with R_Z(MEAN); i.e., the mean cycle curve, based on cycles 8 through 20. It is seen that while the mean cycle curve peaks at t = 50 with a value near 114, cycle 21 peaks much earlier at t = 39 with a value near 188. Except for a few instances, $R_Z(SCN 21) \ge R_Z(MEAN)$. Cycle 21 is now definitely in decline and the $R_Z(PROVISIONAL)$ numbers for January 1983 (t = 79) indicate \overline{R}_{13} value near 83 ± 15 [based on equation (12); Table 1]. \overline{R}_{13} (SCN 21) is shown in Figure 25. Values of \overline{R}_{13} (SCN 21) are plotted, based on R_Z(FINAL) values through June 1981. Superposition of Figures 24 and 25 reveal that, without a doubt, cycle 21 is steadily declining. "Bumps" may be beginning to appear in the \overline{R}_{13} (SCN 21) curve, reminiscent of cycle 20. These bumps represent enhanced periods of sunspot number and for cycle 20 they revealed a "two-faced" Sun, having one hemisphere which was very active and one hemisphere which was very quiet. Large coronal holes were apparent on the Sun during the decline of cycle 20. Perhaps, cycle 21 is entering a similar "active-inactive" time period, complete with large coronal holes.

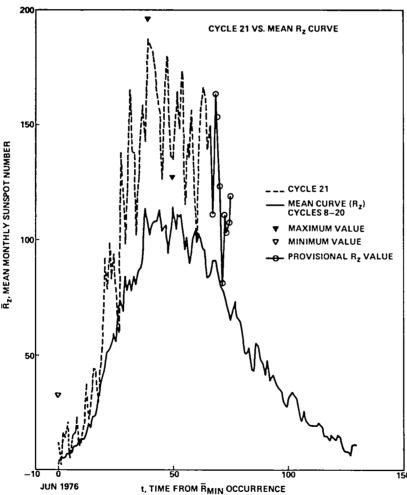


Figure 24. Monthly mean sunspot number for cycle 21 compared to mean monthly sunspot number based on cycles 8 through 20.

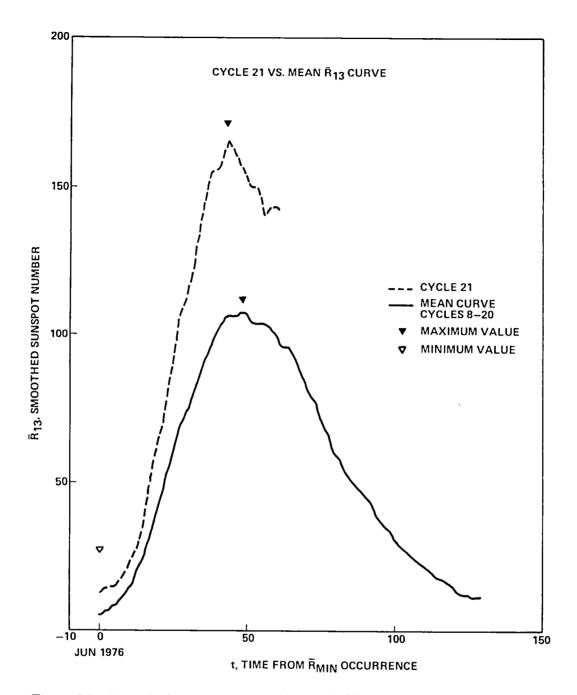


Figure 25. Smoothed sunspot number for cycle 21 compared to mean smoothed sunspot number based on cycles 8 through 20.

In Figure 26, \overline{R}_{13} (SCN 21) is compared with \overline{R}_{13} (HIGH- \overline{R}_{MAX}) and \overline{R}_{13} (LOW- \overline{R}_{MAX}). Cycle 21 is observed to lie above the mean \overline{R}_{13} curve for HIGH- \overline{R}_{MAX} cycles. In Figure 27, \overline{R}_{13} (SCN 21) is compared with \overline{R}_{13} (LONG-PERIOD) and \overline{R}_{13} (SHORT-PERIOD). Cycle 21 is observed to lie above the mean \overline{R}_{13} curve for SHORT-PERIOD cycles. Together, these curves suggest that cycle 21 can be categorized as a HIGH- \overline{R}_{MAX} cycle and probably as a SHORT-PERIOD cycle. If so, then Φ_t (SCN 21) at \overline{R}_{MAX} , on the basis of equation (18) and Figure 12, equals 0.34 implying that MIN-MIN PERIOD (SCN 21) equals about 122 ± 5 months (the approximate range of observed SHORT-PERIOD cycles); thus, cycle 21 may end about August 1986 ± 5 months.

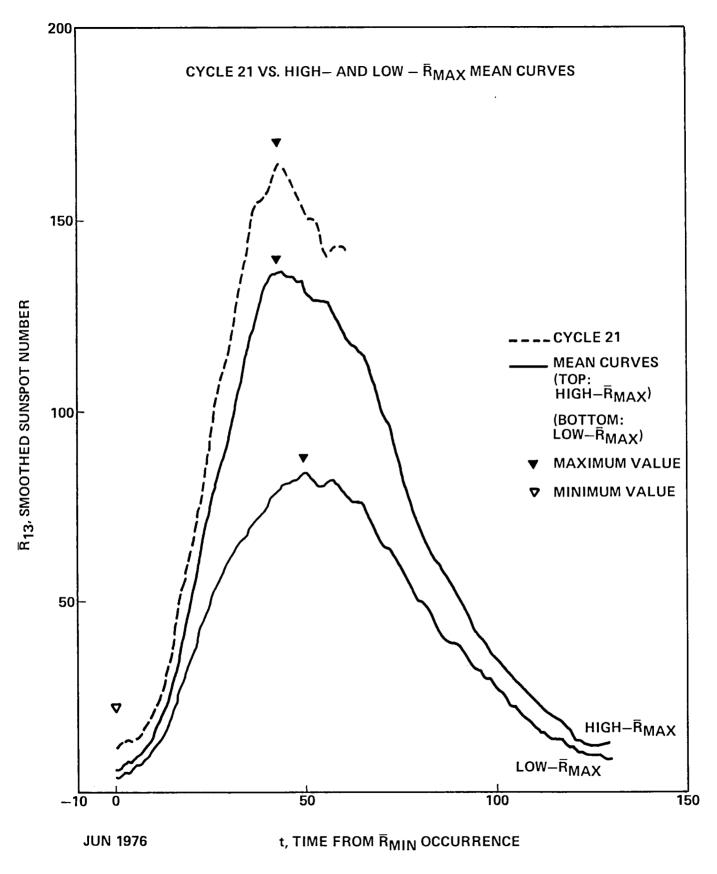


Figure 26. Smoothed sunspot number for cycle 21 compared to mean smoothed sunspot number based on HIGH- and LOW- \bar{R}_{MAX} cycles.

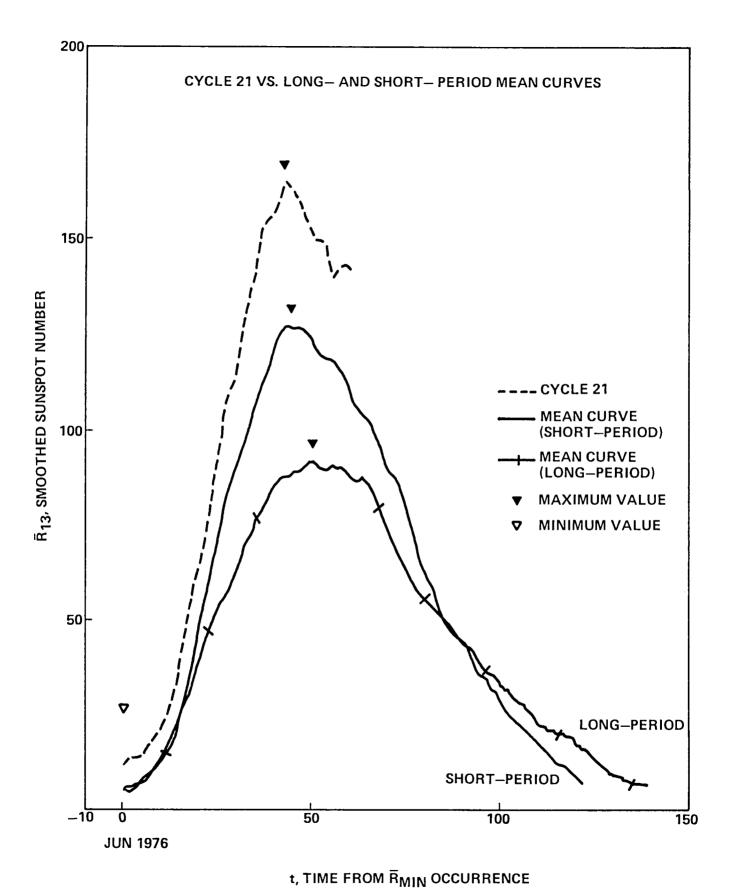


Figure 27. Smoothed sunspot number for cycle 21 compared to mean smoothed sunspot number based on LONG- and SHORT-PERIOD cycles.

Comparison of observed $R_Z(t)_{SCN}$ and/or $\overline{R}_{13}(t)_{SCN}$ values with mean values (and their standard deviations as a function of t) may offer a means whereby values for R_Z and/or \overline{R}_{13} for a cycle can accurately be predicted a year or more ahead of time. That is, if Q_Z and/or Q_{13} is monitored as a function of t where

$$Q_{Z} = \frac{R_{Z}(t)_{SCN} - R_{Z}(t)_{COMPARISON MEAN}}{{}^{S}Z^{(t)}_{COMPARISON MEAN}}$$
(82)

and

$$Q_{13} = \frac{\overline{R}_{13}(t)_{SCN} - \overline{R}_{13}(t)_{COMPARISON MEAN}}{s_{13}(t)_{COMPARISON MEAN}}$$
 (83)

 $R_Z(t)$ and $\overline{R}_{13}(t)$ can be estimated for a cycle on the basis of trend analysis. Q_Z and Q_{13} are merely residuals in terms of standard deviation units as a function of t. The terms $R_Z(t)_{COMPARISON\ MEAN}$ and $\overline{R}_{13}(t)_{COMPARISON\ MEAN}$ refer to the $R_Z(t)$ and $\overline{R}_{13}(t)$ values for a particular comparison group; i.e., the MEAN cycle group (cycles 8 through 20), the HIGH- or LOW- \overline{R}_{MAX} cycle group, or the LONG- or SHORT-PERIOD cycle group. Similarly, the terms $S_Z(t)_{COMPARISON\ MEAN}$ and $S_{13}(t)_{COMPARISON\ MEAN}$ refer to the $S_Z(t)$ or $S_{13}(t)$ values for the same comparison groups. Values for these comparison groups are given in Table 8. Again, equations (82) and (83) simply tell us how many standard deviation units that a particular cycle is above (when Q_Z or Q_{13} is greater than 0) or below (when Q_Z or Q_{13} is less than 0) the mean of the comparison group (as a function of t). An example for cycle 21 is given below.

Based on the MEAN cycle group, Q_Z and Q_{13} are computed for cycle 21 in Table 16 and Q_{13} is plotted as a function of t in Figure 28. We see that Q_{13} began at initially high value ($Q_{13} \sim 2.5$) and decreased during the first 20 months of the cycle to a value which has remained fairly stable for the past 3.5 years. The mean and standard deviation of Q_{13} for all observed t (i.e., $0 \le t \le 61$) are $\overline{x} = 1.61$ and s = 0.39, respectively. However, for $t \ge 20$, $\overline{x} = 1.39$ and s = 0.08. If the present trend continues, then \overline{R}_{13} can be projected for future t in cycle 21 by means of equation (83), rewritten and solving for \overline{R}_{13} below as

$$\overline{R}_{13}(t) = Q_{13} s_{13} (t)_{MEAN} + \overline{R}_{13} (t)_{MEAN}$$
 (84)

Since $Q_{13} \sim 1.39 \pm 0.08$, \overline{R}_{13} may be approximated to within about 6 percent. Continued monitoring of Q_{13} is required for an accurate assessment of future \overline{R}_{13} values. Similar examples could be worked for the other comparison groups, but have not been here, since comparison to the MEAN cycle group will suffice.

TABLE 16. COMPARISON OF CYCLE 21 AND MEAN OF CYCLES 8 THROUGH 20 IN TERMS OF $\rm R_Z,\,Q_Z,\,\bar{R}_{13},\,AND\,\,Q_{13}$

						15	13		
		MEAN OF	CYCLES 8-20			CYC	LE 21		
t	Rz	s _z (t)	R₁3	s ₁₃ (t)	Rz(SCN)	Q_z	R ₁₃ (SCN)	Q ₁₃	NOTES
-		-2		-13,-7		-2	10	-13	
0	4.08	2.99	5.18	2.74	12.2	2.72	12.2	2.56	JUNE 1976 (R _{MIN})
1	4.33	4.23	5.33	2.81	1.9	-0.57	12.9	2.69	in the second
2	4.78	5.10	5.83	2.92	16.4	2.28	14.0	2.80	
3	5.38	6.11	6.45	2.88	13.5	1.33	14.3	2.73	
4	6.65	5.03	7.07	2.74	20.6	2.77	13.5	2.35	
5	5.15	4.22	7.65	2.69	5.2	0.01	13.5	2.17	
6	6.58	4.15	8.47	2.93	15.3	2.10	14.8	2.16	
7	10.32	6.09	9.62	3.21	16.4	1.00	16.7	2.21	
8	10.32	5.08	10.82	3.41	23.1	2.54	18.1	2.13	
9	12.88	6.81	12.12	3.71	8.7	-0.61	20.0	2.12	
10	12.88	6.60	13.60	4.12	12.9	0.00	22.2	2.09	
11	12.58	7.11	15.43	4.71	18.6	0.85	24,2	1.86	
12	15.75	8.99	17.59	5.31	38.5	2.53	26.3	1.64	
13	19.92	9.25	19.85	6.00	21.4	0.16	29.0	1.53	
14	18.22	9.12	22.46	7.19	30.1	1.30	33.4	1.52	
15	23.35	9.28	25.45	8.11	44.0	2.23	39.1	1.68	
16	23.98	11.10	28.70	8.97	43.8	1.79	45.6	1.88	
17	31.89	14.60	32.16	10.48	29.1	-0.19	51.9	1.88	
18	31.72	14.04	35.75	12.06	43.2	0.82	56.9	1.75	
19	39.41	19.65	39.05	13.58	51.9	0.64	61.3	1.64	
20	43.56	19.67	42.55	15.10	93.6	2.54	64.5	1.45	
21	51.36	12.29	46.74	16.71	76.5	2.05	69.6	1.37	
22	52.47	27.31	50.89	18.60	99.7	1.73	76.9	1.40	
23	55.77	25.72	54.95	20.72	82.7	1.05	83.2	1.36	
24	58.74	26.56	59.05	22.58	95.1	1.37	89.3	1.34	
25	56.28	28.27	62.78	24.26	70.4	0.50	97.4	1.43	
26	65.67	25.17	66.02	26.11	58.1	-0.30	104.0	1.45	
27	76.72	28.03	68.82	27.92	138.2	2.19	108.4	1.42	
28	69.47	34.61	71.38	28.76	125.1	1.61	111.1	1.38	
29	84.18	41.03	73.88	29.06	97.9	0.33	113.3	1.36	
30	77.99	32.11	76.42	30.00	122,7	1.39	117.7	1.38	
31	82.48	39.42	79.38	31.21	166.6	2.13	123.7	1,42	
32	78.28	38.82	83.10	32.85	137.5	1,53	130.9	1.46	
33	84.22	32.83	86.39	34.75	138.0	1.64	136.5	1.44	
34	81.10	26.04	89.09	35.78	101.5	0.78	141.1	1.45	
35	87.06	32.46	91.48	36.38	134.4	1.46	147.3	1.53	
36	88.31	42.69	93.77	37.84	149.5	1.43	153.0	1.57	
37	98.13	49.95	96.04	39.22	159.4	1.23	155.0	1.50	
38	113.14	50.05	98.57	40.52	142.2	0.58	155.4	1.40	
39	108.06	52.10	100.87	41.42	188.4	1.54	155.7	1.32	SEP 1979
									OC: 1373
40	103.02	37.33	102.67	41.48	186.2	2.23	157.8	1.33	
41	107.46	51.31	104.47	41.26	183.2	1.48	162.3	1.40	= .
42	108.34	55,33	105.50	40.90	176.3	1.23	164.5	1.44	DEC 1979 (R _{MAX})
43	108.33	49.32	105.97	40.12	159.6	1.04	163.9	1.44	
44	113.28	52.52	106.22	39.11	155.0	0.79	162.6	1.44	
45 40	103.93	39.92	106.07	37.92	126.2	0.56	160.9	1.45	
46	104.91	27.25	106.22	37.48	164.1	2.17	158.7	1.40	
47	106.36	34.32	106.65	37.30	179.9	2.14	156.3	1.33	
48	93.88	41.66	106.87	35.96	157.3	1.52	154.7	1.33	
49	103.75	45.34	106.52	34.29	136.3	0.72	152.8	1.35	
50	113.52	44.54	105.55	32.86	135.4	0.49	150.3	1.36	
51	103.77	46.38	104.45	32.52	155.0	1.10	150.1	1.40	
52	111.02	39.89	103.40	33.10	164.7	1.35	150.2	1.41	
53	109.76	42.36	102.75	33.28	147.9	0.90	147.7	1.35	
54	111.36	28.71	102.95	33.12	174.4	2.20	142.7	1.20	
55	97.05	29.93	103.37	32.75	114.0	0.57	140.3	1.13	
56	101.18	35.48	102.88	32.08	141.3	1.13	141.5	1.20	
57	89.54	43.58	102.09	30.93	135.5	1.05	143.0	1.32	
58	94.32	34.02	101.20	30.07	156.4	1.82	143.4	1.40	
59	100.99	32,71	99.52	29.93	127.5	0.81	142.9	1.45	

TABLE 16. (Continued)

		MEAN OF C	YCLES 8-20			CYC	LE 21		
t	R _z	s _z (t)	R 13	s ₁₃ (t)	R _z (SCN)	o _z	R ₁₃ (SCN)	Q ₁₃	NOTES
60	104.35	34.12	97.35	29.70	90.9	-0.39	141.5	1.49	
61	103.46	31.86	96.02	29.59	143.8	1.27			
62	101.78	28.76	95.32	29.16	158.7	1.98			
63	96.85	26.17	94.95	28.25	167.3	2,69			
64	96.15	38.71	94.63	27.38	162.4	1.71			
65	84.28	38.92	93.38	26.47	137.5	1.44			
66	85.31	33.38	91.41	25.68	150. 1	1.94			
67	90.85	32.61	89.04	25.15					
68	90.98	25.50	86.41	24.90					
69	90.76	33.64	83.72	24.67					
70	85.21	23.55	81.32	24.01					
71	79.83	27.92	79.71	22.74					
72	78.48	23.38	78.44	21.50					
73	72.82	23.88	76.58	19.94					
74	68.92	25.98	74.20	18.32					
75	65.45	27.03	71.25	16.91					
76	69.48	25.33	68.17	15.58					
7 7	72.52	25.42	65.62	14.82					
78	66.82	18.69	63.08	14.16					
79	64.61	15.52	60.45	13.50					
80	60.25	17.24	58.66	13:05					
81	50.63	18.62	57.62	12.74					
82	51.11	12.40	56.33	12.56					
83	52.84	19.32	54.38	12.65					
84	44.42	14.93	52.27	12.85					
85	43.88	14.29	50.58	13.38					
86	54.92	23.47	49.03	14.22					
87	54.43	22.88	47.73	14.67					
88	49.01	23.82	46.88	15.07					
89	46.54	17.61	45.85	15.55					
90	42.37	18.27	44.88	15.64		•			
91	48.46	24.61	44.12	15.37					
92	39.08	19.39	42.71	14.85					
93	40.48	18.41	40.55	14,46					
94	41.27	20.02	38.52	14.30					
95	38.07	18.61	36.99	14,34					
96	35.56	15.47	36.08	14.65					
97	34.81	14.54	35.05	14.75					
98	29.81	15.34	34.05	14.49					
99	27.98	18.43	33.02	14,19					
100	26.58	12.18	31.46	13.92					
101	32.48	18.41	30.12	13.64					
102	34.22	21.44	29.05	13.55					
103	32.24	17.91	27.83	13.46					
104	31.21	15.92	26.78	13.28					
105	23.50	15.13	25.92	13.05					
106	21.05	13.31	25.22	13.03					
107	26.22	17.89	24.37	12.89					
108	21.32	12.81	23.25	12.41					
109	19.81	14.44	22.07	11.96					
110	19.28	13.06	20.84	11,84					
111	18.55	17.43	19.84	12.07					
112	19.28	13.40	19.15	12.13					
113	18.95	13.22	18.36	11.96					
114	20.19	13.80	17.56	11.93					
115	18.62	12.77	17.02	12.13					
116	15.48	12.44	16.55	12.09					
117	15.21	16.48	16.17	11.75					
118	13.12	11,25	15.56	11.25					
119	14.73	12.62	14.67	10.82					

TABLE 16. (Concluded)

		MEAN OF C	YCLES 8-20						
t	Rz	s _z (t)	R ₁₃	s ₁₃ (t)	R _z (SCN)	Q _z	R ₁₃ (SCN)	o ₁₃	NOTES
120	14.06	14.80	13.72	10.27					
121	13.37	14.13	12.75	9.65					
122	14.78	10.64	12.10	9.04					
123	13.72	10.83	11.82	8.31					
124	9.39	7.50	11.65	7.76					
125	7.91	7.07	11.52	7.27					
126	8.38	5.26	11.28	6.44					
127	6.83	4.97	11.05	5.56					
128	11.84	6.41	10.75	5.16					
129	12.15	8.59	10.62	5.08					

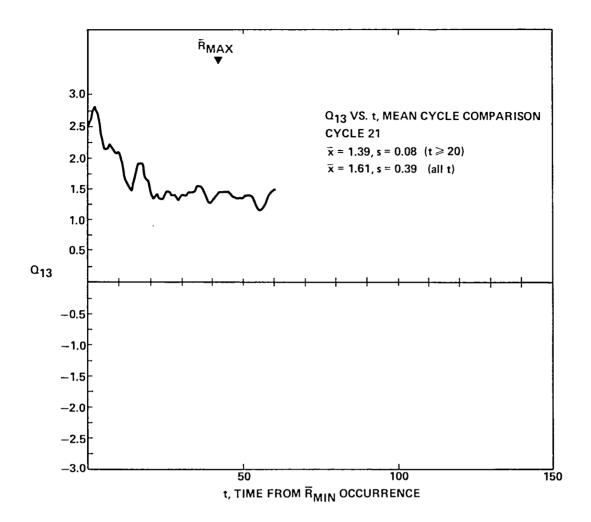


Figure 28. Q₁₃ value versus time from cycle 21 minimum smoothed sunspot number occurrence.

It should be noted that Q_{13} , in the above example, should not increase with large t, since doing so will cause \overline{R}_{13} to be much higher than has ever been observed at $\overline{R}_{MIN}(SCN+1)$ occurrence. If $Q_{13}=1.4$ is valid at t=120, then $\overline{R}_{13}=28.1$, about a factor of 2 higher than the mean value of \overline{R}_{13} for t=120. If cycle 21 is a short-period cycle, as it would seem to be, then Q_{13} must become smaller or perhaps even negative in value to reduce the \overline{R}_{13} computed value at large t. [If cycle 21 is a SHORT-PERIOD cycle, then a value of $\overline{R}_{MIN}(SCN 22)=8$ at t=122 implies that $Q_{13}\sim -0.45$; a value of $\overline{R}_{MIN}(SCN 22)=16$ implies that $Q_{13}\sim 0.43$.)]

Figure 29 ends this subsection by comparing observed \overline{R}_{13} values for cycle 21 with the observed SLOPE_{ASC} and predicted SLOPE_{DES} lines [based on equation (22)]. It is seen that SLOPE_{ASC} reasonably matches \overline{R}_{13} (SCN 21) values for $t \ge 24$ and that SLOPE_{DES} matches at least through t = 55. SLOPE_{DES} suggests that cycle 21 will end no later than May 1987 and no earlier than June 1986, since computed values for \overline{R}_{13} are 0 and about 20, respectively.

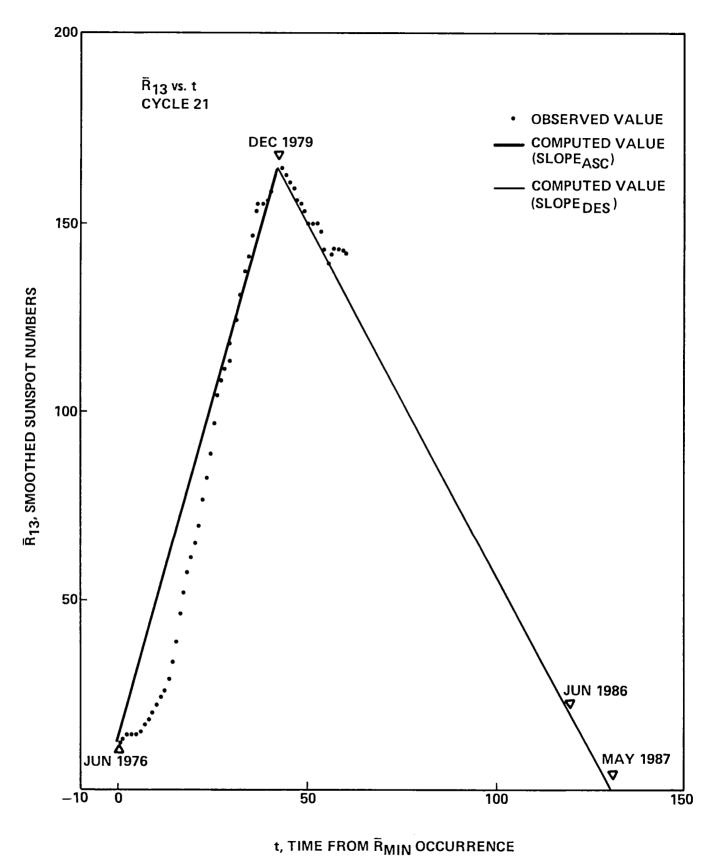


Figure 29. Comparison of cycle 21 smoothed sunspot number (versus time) with computed ascent slope and projected descent slope.

V. CONCLUSIONS

Based on cycles 8 through 20 and using a schematic approach, values for a number of cycle parameters have been compared and their mean values, standard deviations, and frequency of occurrence histograms computed. Some of these parameters were plotted as a function of solar cycle number (SCN), and trends were determined. Also, linear regression analysis was performed relating late cycle parameters with early cycle parameters, in the hopes of finding any early cycle parameters which might be of use for providing good estimates for later occurring cycle parameters. It is found that \overline{R}_{MIN} , $\frac{\Delta}{GPV}$ \overline{R}_{13} and 24

 $\sum_{t=0}^{24} R_Z(t)$, especially the latter two, appear to be good candidate early cycle parameters for forecasting

values of later occurring cycle parameters.

In addition to the "mean" cycle, it is noted that cycles may be divided into groups based on values of \overline{R}_{MAX} and MIN-MIN PERIOD (especially the latter) or, more simply, cycle duration, and that these groupings allow better approximations to selected cycle parameters. Further, it is noted that, usually, cycles of low- \overline{R}_{MIN} value are cycles of low- \overline{R}_{MAX} , long-ASC, and long-MIN-MIN PERIOD values; cycles of high- \overline{R}_{MIN} value are, usually, cycles of high- \overline{R}_{MAX} , short-ASC, and short-MIN-MIN PERIOD values. Means and standard deviations for the subgroups HIGH- \overline{R}_{MAX} , LOW- \overline{R}_{MAX} , LOW- \overline{R}_{MAX} , LOW- \overline{R}_{MAX} , LONG-PERIOD cycle, and SHORT-PERIOD cycle were computed, and it is noted that cycle 21 is observed (and could have been predicted) to be a HIGH- \overline{R}_{MAX} cycle. Furthermore, cycle 21 appears to also be a SHORT-PERIOD cycle. On this basis, we expect cycle 21 to end during the latter part of 1986 or very early part of 1987. A continuation of this study, based on an empirical curve fit of observed \overline{R}_{MAX} values versus SCN (8 through 20), appears in Wilson [15].

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APPENDIX A

DATE	t	R _z	R ₁₃	Τ	$\triangle \overline{R}_{13}$	Φ_{t}	Φ_{T}	NOTES
NOV 1833	0 12345678910112314567899111231456178192122222223333333333333333333333333333	5.9 9.9 4.9 18.1 3.9 1.4 8.8 7.8 8.7 4.0 11.5 24.8 30.5 34.5 7.5 24.5 19.7 61.5 43.6 33.2 59.0 100.8 95.2 100.0 77.5 88.6 107.6 11.4 124.7 116.7 116.7 117.8 117.4 120.9 117.6 11	7.3 7.4 7.8 7.8 7.7 8.4 10.2 12.2 13.4 13.7 14.6 17.8 21.8 24.3 27.5 31.9 37.9 44.6 50.4 55.1 60.2 67.1 73.8 80.5 86.7 93.3 99.5 103.9 105.7 107.2 109.9 116.1 125.6 136.9 138.2 139.0 139.4 142.7 145.8 146.9 146.4 145.2 141.5	-40 -39 -38 -37 -36 -37 -36 -37 -38 -37 -38 -37 -38 -38 -38 -38 -38 -38 -38 -38 -38 -38	+0.1 +0.4 0.0 -0.1 +1.8 0.0 -1.7 8 +1.2 +1.2 +1.2 +1.2 +1.3 +1.3 +1.3 +1.3 +1.3 +1.3 +1.3 +1.3	0.00 .01 .02 .03 .03 .04 .05 .06 .07 .08 .09 .09 .10 .11 .12 .13 .14 .15 .16 .17 .18 .19 .20 .21 .22 .22 .23 .24 .25 .26 .27 .28 .29 .30 .31 .32 .33 .34 .35 .36 .37 .38 .39 .30 .30 .30 .30 .30 .30 .30 .30	0.55 .56 .57 .58 .59 .60 .61 .63 .64 .65 .66 .67 .72 .73 .74 .75 .76 .77 .78 .80 .81 .82 .83 .84 .85 .88 .90 .91 .92 .93 .94 .97 .98 .99 .99 .99 .99 .99 .99 .99 .99 .99	R _{MIN} SCN 8
	44 45 46 47	162.8 134.0 96.3 123.7	136.5 130.9 127.4 127.2	4 5 6 7	-5.6 -3.5 -0.2 +0.6	.38 .39 .40 .41	.03 .04 .05 .05	

107.0 129.8 144.8 140.8 140.8 126.6 137.6 108.2 77.8 107.5 77.8 107.5 77.8 107.5 132.7 132.7 131.2 132.7 131.2 132.7 131.2 132.7 131.2 132.7 133.8 103	127.8 126.2 121.3 116.7 113.5 111.2 108.6 105.2 101.6 100.8 98.9 93.6 87.4 82.2 79.6 80.8 85.4 87.9 87.5 86.5 82.0 81.7 77.6 72.1 68.2 66.1 65.1 65.1 65.1 65.1 65.1 65.1 65.1	8 9 10 11 13 14 15 16 17 18 19 20 21 22 22 22 22 23 31 33 33 33 34 44 44 45 46 47 48 49 50 51 51 52 53 53 54 54 54 54 54 54 54 54 54 54 54 54 54	-1.6962364689322626540870384215910880602914751757-2.6.7	.42 .43 .445 .447 .456 .553 .556 .556 .559 .661 .666 .667 .772 .778 .778 .778 .779 .779 .779 .779 .779	.06 .07 .08 .08 .09 .10 .11 .12 .13 .14 .15 .16 .17 .18 .19 .20 .21 .22 .23 .24 .25 .27 .27 .28 .29 .30 .31 .31 .32 .33 .34 .35 .36 .37 .37 .37 .37 .37 .37 .37 .37 .37 .37	ΔR R 13
67•4 55•7	39.5 37.4	50 51	-2.1 -0.7	•78 •78	.38 .39	
	129.8 144.9 84.8 140.6 137.6 108.2 78.6 90.8 77.4 102.5 77.8 90.8 77.8 90.8 61.8 63.6 63.6 63.7 63.8 63.7 63.8 63.7 63.8 63.7 63.8 63.7 63.8 63.7 63.8 63.7 63.8 63.7 63.8 63.7 63.8 63.7 63.8 63.8 63.7 63.8 63.8 63.7 63.8	129.8 126.2 144.9 121.3 84.8 116.7 140.8 113.5 126.6 111.2 137.6 108.6 94.5 105.2 108.2 101.6 78.8 100.8 73.6 98.9 90.8 93.6 77.4 87.4 79.8 82.2 107.6 79.6 102.5 80.8 77.7 85.4 61.8 87.9 53.8 87.5 54.6 86.5 84.7 84.7 131.2 83.0 132.7 82.0 90.8 81.7 68.8 82.9 81.2 81.7 77.6 67.8 63.6 82.9 81.7 77.6 67.8 72.1 65.9 66.1 48.5 65.1 60.7 62.3 57.8 57.5 74.0 53.5	129.8 126.2 9 144.9 121.3 10 84.8 116.7 11 140.8 113.5 12 126.6 111.2 13 137.6 108.6 14 94.5 105.2 15 108.2 101.6 16 78.8 100.8 17 73.6 98.9 18 90.8 93.6 19 77.4 87.4 20 79.8 82.2 21 107.6 79.6 22 102.5 80.8 23 77.7 85.4 24 61.8 87.9 25 53.8 87.5 26 54.6 86.5 27 84.7 84.7 28 131.2 83.0 29 132.7 82.0 30 90.8 81.7 31 68.8 82.9 33 81.7 34 34 77.6 35 35 <td< td=""><td>129.8 126.2 9 -4.9 144.9 121.3 10 -4.6 84.8 116.7 11 -3.2 140.8 113.5 12 -2.3 126.6 111.2 13 -2.6 137.6 108.6 14 -3.4 94.5 105.2 15 -3.6 108.2 101.6 16 -0.8 78.8 100.8 17 -1.9 73.6 98.9 18 -5.3 90.8 93.6 19 -6.2 77.4 87.4 20 -5.2 79.8 82.2 21 -2.6 107.6 79.6 22 +1.2 102.5 80.8 23 +4.6 77.7 85.4 24 +2.5 61.8 87.9 25 -0.4 53.8 87.5 26 -1.0 54.6 86.5 27 -1.8 84.7 84.7 28 -1.7 131.2 83.0 29</td><td>129.8 126.2 9 -4.9 .42 144.9 121.3 10 -4.6 .43 84.8 116.7 11 -3.2 .44 140.8 113.5 12 -2.3 .45 126.6 111.2 13 -2.6 .46 137.6 108.6 14 -3.4 .47 94.5 105.2 15 -3.6 .47 108.2 101.6 16 -0.8 .48 78.8 100.8 17 -1.9 .49 73.6 98.9 18 -5.3 .50 90.8 93.6 19 -6.2 .51 77.4 87.4 20 -5.2 .52 79.8 82.2 21 -2.6 .53 100.5 80.8 23 +4.6 .54 77.7 85.4 24 +2.5 .55 61.8 87.9 25 -0.4 .56 53.8 87.5 26 -1.0 .57 54.6 86.5</td></td<> <td>129.8 126.2 9 -4.9 .42 .07 144.9 121.3 10 -4.6 .43 .08 84.8 116.7 11 -3.2 .44 .08 140.8 113.5 12 -2.3 .45 .09 126.6 111.2 13 -2.6 .46 .10 137.6 108.6 14 -3.4 .47 .11 94.5 105.2 15 -3.6 .47 .11 108.2 101.6 16 -0.8 .48 .12 78.8 100.8 17 -1.9 .49 .13 73.6 98.9 18 -5.3 .50 .14 90.8 93.6 19 -6.2 .51 .15 77.4 87.4 20 -5.2 .52 .15 79.8 82.2 21 -2.6 .53 .16 107.6 79.6 22 +1.2 .53</td>	129.8 126.2 9 -4.9 144.9 121.3 10 -4.6 84.8 116.7 11 -3.2 140.8 113.5 12 -2.3 126.6 111.2 13 -2.6 137.6 108.6 14 -3.4 94.5 105.2 15 -3.6 108.2 101.6 16 -0.8 78.8 100.8 17 -1.9 73.6 98.9 18 -5.3 90.8 93.6 19 -6.2 77.4 87.4 20 -5.2 79.8 82.2 21 -2.6 107.6 79.6 22 +1.2 102.5 80.8 23 +4.6 77.7 85.4 24 +2.5 61.8 87.9 25 -0.4 53.8 87.5 26 -1.0 54.6 86.5 27 -1.8 84.7 84.7 28 -1.7 131.2 83.0 29	129.8 126.2 9 -4.9 .42 144.9 121.3 10 -4.6 .43 84.8 116.7 11 -3.2 .44 140.8 113.5 12 -2.3 .45 126.6 111.2 13 -2.6 .46 137.6 108.6 14 -3.4 .47 94.5 105.2 15 -3.6 .47 108.2 101.6 16 -0.8 .48 78.8 100.8 17 -1.9 .49 73.6 98.9 18 -5.3 .50 90.8 93.6 19 -6.2 .51 77.4 87.4 20 -5.2 .52 79.8 82.2 21 -2.6 .53 100.5 80.8 23 +4.6 .54 77.7 85.4 24 +2.5 .55 61.8 87.9 25 -0.4 .56 53.8 87.5 26 -1.0 .57 54.6 86.5	129.8 126.2 9 -4.9 .42 .07 144.9 121.3 10 -4.6 .43 .08 84.8 116.7 11 -3.2 .44 .08 140.8 113.5 12 -2.3 .45 .09 126.6 111.2 13 -2.6 .46 .10 137.6 108.6 14 -3.4 .47 .11 94.5 105.2 15 -3.6 .47 .11 108.2 101.6 16 -0.8 .48 .12 78.8 100.8 17 -1.9 .49 .13 73.6 98.9 18 -5.3 .50 .14 90.8 93.6 19 -6.2 .51 .15 77.4 87.4 20 -5.2 .52 .15 79.8 82.2 21 -2.6 .53 .16 107.6 79.6 22 +1.2 .53

	99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116	22.1 21.7 26.9 24.9 20.5 12.6 26.5 18.5 38.1 40.5 17.6 13.3 3.5 8.2 8.8 21.1 10.5 9.5	25.4 24.1 23.8 25.1 25.1 23.9 22.8 21.5 20.2 19.3 18.7 18.1 17.4 16.2 14.2 12.0 10.9 10.5	59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75	-1.3 -0.3 +1.3 0.0 -1.2 -1.1 -1.3 -1.3 -0.9 -0.6 -0.6 -0.7 -1.2 -2.0 -2.2 -1.1 -0.4 +0.3	.85 .86 .87 .88 .89 .90 .91 .91 .92 .93 .94 .95 .96 .97 .97	.45 .46 .47 .48 .49 .50 .51 .52 .53 .54 .56 .56 .57	
JUL 1843	0 123456789 10112314 1561789 20122324 25627899 31	9.5 11.8 4.2 5.3 19.1 12.7 9.4 14.6 20.8 12.0 21.5 21.6 23.9 21.5 21.6 25.7 43.8 56.9 47.8 31.1 30.6 47.8 31.1 30.7 43.7 43.7 43.7 43.7 43.7 43.7 43.7 43	10.5 10.8 11.5 12.2 12.3 11.7 11.9 12.9 13.5 14.6 15.7 17.6 20.7 25.7 28.4 29.9 30.6 31.9 33.7 35.7 38.5 40.6 41.5 42.6 44.0 45.0 46.9 49.0 50.6	-554 -552 -552 -559 -445 -445 -445 -445 -445 -445 -445	+0.3 +0.7 +0.7 +0.1 -0.6 +0.2 +1.0 +0.8 +0.3 0.0 +1.1 +1.9 +2.4 +2.7 +3.0 +2.7 +1.5 +0.7 +1.3 +1.8 +2.0 +2.8 +2.1 +0.9 +1.1 +1.0 +1.0 +1.0 +1.0 +1.0 +1.0 +1.0	0.00 .01 .01 .02 .03 .04 .05 .05 .06 .07 .08 .09 .09 .10 .11 .12 .13 .14 .15 .15 .16 .17 .17 .18 .19 .20 .21	0.58 .59 .60 .61 .62 .63 .63 .64 .65 .66 .67 .68 .69 .70 .71 .72 .73 .74 .75 .76 .76 .77 .78 .79 .80 .81 .82	R _{MIN} SCN 9

FEB 1848	32 63.9 33 69.2 34 59.9 35 65.1 36 46.5 37 54.8 38 107.1 39 55.9 40 60.4 41 65.5 42 62.6 43 44.9 44 85.7 45 44.7 46 75.4 47 85.3 48 52.2 49 140.6 50 161.2 51 180.4 52 138.9 53 109.6 54 159.1 55 111.8 56 108.9 57 107.1 58 102.2	54.8 58.6 60.1 61.3 62.5 63.2 63.9 63.8 63.4 64.9 66.0 69.8 75.6 83.1 91.5 96.6 102.5 109.3 113.0 116.6 120.3 123.3 128.7 132.0 129.1 124.6 121.6	-23 -22 -21 -20 -19 -18 -17 -16 -15 -13 -12 -11 -10 - 8 - 7 - 6 - 5 - 4 - 3 - 2 - 1 - 1 - 2 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3 - 3	+3.8 +1.5 +1.2 +1.2 +0.7 -0.1 -0.4 +1.5 +1.1 +3.8 +7.5 +4.1 +5.9 +6.8 +3.7 +3.7 +3.7 +3.8 +3.7 +3.6 +3.6 +3.6 +3.6 +3.6 +3.6 +3.6 +3.6	.21 .22 .23 .24 .25 .26 .27 .28 .29 .30 .31 .32 .33 .34 .35 .36 .37 .38 .39 .39 .39 .39	.82 .83 .84 .85 .85 .86 .87 .88 .89 .90 .91 .92 .93 .94 .95 .96 .97 .98 .99 0.00 .01	ĠPV ^R 13
	59 129.0 60 139.2 61 132.5 62 100.3 63 132.4 64 114.6 65 159.9 66 156.7 67 131.7 68 96.5 69 102.5 70 80.6 71 81.2 72 78.0 73 67.7 74 93.7 75 71.5 76 99.7 77 97.0 78 78.0 79 89.4 80 82.6 81 44.1 82 61.6	122.7 124.7 125.4 125.7 125.0 123.9 121.0 116.5 111.2 108.2 105.4 102.3 99.0 93.1 88.1 85.7 79.5 78.2 76.2 74.3 73.7 73.4 71.5	4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 27	+2.0 +0.7 +0.3 -0.7 -1.1 -2.9 -4.5 -5.3 -3.0 -2.8 -3.1 -3.3 -5.9 -5.0 -2.4 -3.0 -2.4 -3.0 -2.9 -1.9 -0.6 -0.3 -1.9 -3.4	.40 .40 .41 .42 .43 .44 .45 .46 .47 .48 .49 .50 .51 .52 .53 .55 .55	.03 .04 .05 .06 .06 .07 .08 .09 .10 .11 .12 .13 .13 .14 .15 .15 .16 .17 .17	∆- GNV ^R 13

83 84 85 86 87 88 89 90 91 92	70.0 39.1 61.6 86.2 71.0 54.8 61.0 75.5 105.4 64.6 56.5	68.1 66.5 67.1 67.0 66.8 67.3 67.1 66.7 66.4 65.4	28 29 30 31 32 33 34 35 36 37 38	-1.6 +0.6 -0.1 -0.2 +0.5 -0.2 +0.6 -0.3 -1.0 -1.1	.56 .56 .57 .58 .59 .60 .61	.19 .20 .21 .22 .22 .23 .24 .24 .25 .26
94	62.6	63.8	39	+0.3	.63	.27
95	63.2	64.1	40	+0.1	.64	.28
96	36.1	64.2	41	-1.9	.64	.28
97 98 99	57.4 67.9 62.5	62.3 60.5 60.7	42 43 44	-1.8 +0.2 +0.1	.65 .66	.29 .30 .31
100	50.9	60.8	45	-1.0	•67	.31
101	71.4	59.8	46	-0.4	•68	.32
102 103 104	68.4 66.4 61.2	59.4 58.9 56.9	47 48 49	-0.5 -2.0 -1.1	.68 .69 .70	.33 .33
105	65.4	55.8	50	+0.3	.70	.35
106	54.9	56.1	51	-0.9	.71	.35
107	46.9	55.2	52	-2.2	.72	.36
108	42.0	53.0	53	-2.1	.72	.37
109	39.7	50.9	54	-2.0	.73	.38
110	37.5	48.9	55	-1.7	.74	.38
111	67.3	47.2	56	-1.6	• 74	.39
112	54.3	45.6	57	-1.1	• 75	.40
113	45.4	44.5	58	-0.2	• 76	.40
114	41.1	44.3	59	+0.7	.77	.41
115	42.9	45.0	60	+0.2	.77	.42
116	37.7	45.2	61	-1.2	.78	.42
117	47.6	44.0	62	-2.1	.79	.43
118	34.7	41.9	63	-2.0	.79	.44
119	40.0	39.9	64	-1.9	.80	.44
120	45.9	38.0	65	-2.1	.81	•45
121	50.4	35.9	66	-1.6	.81	•46
122	33.5	34.3	67	-1.6	.82	•47
123	42.3	32.7	68	-1.4	. 83	.47
124	28.8	31.3	69	-1.2	. 83	.48
125	23.4	30.1	70	-1.9	. 84	.49
126	15.4	28.2	71	-2.6	• 85	•49
127	20.0	25.6	72	-1.9	• 85	•50
128 129 130	20.7 26.4 24.0	23.7 22.0 20.8	73 74 75	-1.7 -1.2 -0.1	. 86 . 87 . 87	.51 .51
131	21.1	20.7	76	-0.3	. 88	•53
132	18.7	20.4	77	-0.4	. 89	•53
133	15.8	20.0	78	-0.5	. 89	•54

	134 135 136 137 138 139 140 141 142 143 144 145 146 147 148	22.4 12.7 28.2 21.4 12.3 11.4 17.4 4.4 9.1 5.3 0.4 3.1 0.0 9.7 4.2 3.1	19.5 18.4 16.9 15.6 14.2 12.9 11.4 10.4 9.2 7.5 6.2 5.4 4.5 3.8 3.6 3.2	79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94	-1.1 -1.5 -1.3 -1.4 -1.3 -1.5 -1.0 -1.2 -1.7 -1.3 -0.8 -0.9 -0.7 -0.2 -0.4 +0.1	.90 .91 .91 .92 .93 .93 .94 .95 .95 .97 .97 .98 .99	.55 .56 .57 .58 .59 .60 .61 .62 .63 .64 .65	
DEC 1855	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 22 23 24 25 26 27 28 29 30 31 32 32 32 32 32 32 32 32 32 32 32 32 32	3.1 0.9 0.4 0.2 0.2 0.2 0.3 0.2 0.3 0.3 0.3 0.3 0.3 0.3 0.3 0.3	3.2 3.6 3.9 3.9 3.9 3.9 3.9 3.9 4.1 5.8 6.2 7.6 2.1 1.6 7.7 1.9 2.5 2.5 2.6 3.0 3.0 3.0 3.0 3.0 3.0 3.0 3.0 3.0 3.0	-50 -49 -48 -47 -46 -45 -44 -43 -42 -41 -40 -39 -38 -37 -36 -35 -34 -32 -27 -26 -27 -26 -27 -22 -21 -21 -21 -21 -21 -21 -21 -21 -21	+0.1 +0.3 +0.3 -0.1 +0.3 +0.6 +0.3 +0.4 +1.4 +1.6 +1.2 +2.1 +3.0 +2.5 +2.3 +2.3 +2.3 +2.3 +3.3 +1.7 +1.7 +2.6 +3.1 +3.1 +3.0 +2.1 +3.0 +3.6 +3.3	0.00 .01 .02 .03 .04 .04 .05 .06 .07 .08 .09 .10 .11 .12 .13 .14 .15 .16 .16 .17 .18 .19 .19 .20 .21 .21 .22 .23 .24	0.65 .66 .67 .68 .69 .69 .70 .71 .72 .72 .73 .74 .75 .76 .77 .78 .78 .79 .80 .81 .82 .83 .83 .84 .85 .85 .86 .87 .88	R _{MIN} SCN 10

	34 91.2 35 51.9 36 66.9 37 83.7 38 87.6 39 90.3 40 85.7 41 91.0 42 87.1	67.6 71.7 75.5 78.9 82.6 85.9 87.9 90.8 93.2	-16 -15 -14 -13 -12 -11 -10 - 9 - 8	+4.1 +3.8 +3.4 +3.7 +3.3 +2.0 +2.9 +2.4 +0.6	.25 .26 .27 .27 .28 .29 .30 .30	.89 .90 .90 .91 .92 .93 .94	GPV ^R 13
FEB 1860	43 95.2 44 106.8 45 105.8 46 114.6 47 97.2 48 81.0 49 82.4 50 88.0 51 98.9 52 71.4 53 107.1	93.8 93.7 94.1 93.9 94.0 95.5 97.3 97.9 97.1 95.5	- 7 - 6 - 5 - 4 - 3 - 2 - 1	-0.1 +0.4 -0.2 +0.1 +1.5 +1.8 +0.6 -0.8 -1.6 -1.0 +0.7	.32 .33 .34 .35 .36 .36 .37 .38 .39	.95 .96 .97 .97 .98	\overline{R}_{MAX}
	54 108.6 55 116.7 56 100.3 57 92.2 58 90.1 59 97.9 60 95.6 61 62.3 62 77.8	95.2 94.9 93.7 93.3 94.5 93.6 90.6 88.1 85.8	1 2 3 4 5 6 7 8 9 10 11 12	-0.3 -1.2 -0.4 +1.2 -0.9 -3.0 -2.5 -2.3 -1.3	.40 .41 .42 .43 .44 .44	.03 .04 .05 .06 .06 .07 .08 .09	
	63 101.0 64 98.5 65 56.8 66 87.8 67 78.0 68 82.5 69 79.9 70 67.2 71 53.7 72 80.5	68.1		+0.2 -0.4			[∆] GNV ^R 13
	73 63.1 74 64.5 75 43.6 76 53.7 77 64.4 78 84.0 79 73.4 80 62.5 81 66.6 82 42.0 83 50.6 84 40.9	66.7 65.3 63.7 62.5 60.8 58.5 57.6 58.2 58.6 57.6	24 25 26 27 28 29 30 31 32 33	-1.0 -1.4 -1.6 -1.2 -1.7 -2.3 -0.9 +0.6 +0.4 -1.0 -2.2 -3.5	.55 .56 .57 .58 .59 .60 .61	.19 .20 .21 .21 .22 .23 .24 .25 .25	

92	91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 129 130 131 131 131 131 131 131 131 131 131	40.8 32.7 48.1 22.9 37.7 41.2 57.1 66.8 40.8 57.1 66.8 57.1 66.8 57.6 66.7 33.6 36.7 36.7	44.0 43.0 43.0 44.0 46.6 47.5 46.6 47.5 46.6 47.5 47.5 47.6 47.5 47.6	43 44 45 46 47 49 49 51 52 53 54 55 55 55 55 56 61 62 63 64 56 67 77 77 77 77 77 77 77 77 77 77 77 77	-0.2 -0.8 +0.2 +1.6 +1.2 +0.6 0.0 +0.3 -0.7 -1.5 -1.2 -1.0 -2.1 -1.3 -0.8 -1.2 -1.3 -1.4 -1.8 -1.6 -1.2 -1.8 -	.69 .70 .70 .71 .72 .73 .74 .75 .76 .77 .78 .79 .81 .81 .82 .83 .84 .85 .87 .88 .89 .90 .91 .92 .93 .94 .95 .99 .99	.31 .32 .33 .33 .34 .35 .37 .38 .40 .41 .42 .43 .44 .45 .47 .48 .49 .51 .52 .53 .55 .56 .60 .61 .66 .66 .66 .66 .66 .66 .66 .66 .66
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MAR 1867	0 9.2 1 5.1 2 2.9 3 1.5	5.2 5.3 5.3 6.3	-41 -40 -39 -38	+0.1 0.0 +1.0 +1.6	0.00 .01 .01 .02	0.67 .68 .69 .70	R _{MIN} SCN 11
	4 5.0 5 4.8 6 9.8 7 13.5 8 9.6	7.9 9.2 10.6 12.6 14.9	-37 -36 -35 -34 -33	+1.3 +1.4 +2.0 +2.3 +2.2	.03 .04 .04 .05	.71 .71 .72 .73	
	9 25.2 10 15.6 11 15.8 12 26.5	17.1 19.4 21.6 24.4	-32 -31 -30 -29	+2.3 +2.2 +2.8 +3.5	.06 .07 .08 .09	.75 .75 .76 .77	
	13 36.6 14 26.7 15 31.1 16 29.0 17 34.4	27.9 32.0 35.8 39.5 43.2	-28 -27 -26 -25 -24	+4.1 +3.8 +3.7 +3.7 +2.9	.09 .10 .11 .11	.78 .79 .79 .80	
	17 34.4 18 47.2 19 61.7 20 59.1 21 67.6	46.1 47.4 50.8 57.2	-23 -22 -21 -20	+1.3 +3.4 +6.4 +4.5	.13 .13 .14 .15	. 82 . 83 . 83 . 84	
	22 60.9 23 59.3 24 52.7 25 41.0	61.7 64.8 68.1 69.4	-19 -18 -17 -16	+3.1 +3.3 +1.3 +0.7 +2.3	.16 .16 .17	. 85 . 86 . 87 . 87 . 88	
	26 104.0 27 108.4 28 59.2 29 79.6 30 80.6	70.1 72.4 74.6 77.6 84.4	-15 -14 -13 -12 -11	+2.3 +2.2 +3.0 +6.8 +9.4	.18 .19 .20 .21	.89 .90 .90	△ GPV ^R 13
	31 59.4 32 77.4 33 104.3 34 77.3 35 114.9	93.8 101.8 105.9 110.1 116.2	-10 - 9 - 8 - 7 - 6	+8.0 +4.1 +4.2 +6.1 +5.4	.22 .23 .23 .24 .25	.92 .93 .94 .94	ui v 13
	36 160.0 37 160.0 38 176.0 39 135.6	121.6 127.5 134.1 138.1	- 5 - 4 - 3 - 2	+5.9 +6.6 +4.0 +1.5	.26 .26 .27 .28	.96 .97 .98 .98	
AUG 1870	40 132.4 41 153.8 42 136.0 43 146.4 44 147.5	139.6 140.5 140.2 139.6 138.5	- 1 0 1 2 3	+0.9 -0.3 -0.6 -1.1 -3.1	.28 .29 .30 .30	.99 0.00 .01 .01	\overline{R}_{MAX}
	45 130.0 46 88.3 47 125.3 48 143.2 49 162.4	135.4 132.3 129.3 125.1 120.4	4 5 6 7 8	-3.1 -3.0 -4.2 -4.7 -4.1	.32 .33 .33 .34	.03 .03 .04 .04	

9 11123456789012345678901234567890123456789 1123456789012345678901234567890123456789	-3.4 -2.1 -2.5 -4.8 -4.1 -0.9 -0.6 -0.7 -0.9 -0.1 -0.7 -0.9 -0.1 -0.7 -0.9 -0.1 -0.7	.35 .37 .38 .39 .40 .41 .43 .44 .45 .45 .47 .48 .49 .55 .55 .55 .55 .55 .57 .57 .58 .60 .66 .67 .67 .69 .70 .71	ΔNR 13
145.5 116.3 91.7 112.9 103.0 110.8 110.0 110.3 80.3 107.8 89.0 103.0 105.4 98.9 90.3 98.0 79.5 98.9 120.1 98.3 88.4 99.0 102.1 101.0 107.6 101.9 109.9 101.9 105.5 102.0 92.9 101.7 114.6 101.6 103.5 100.9 112.0 97.4 83.9 92.2 86.7 87.8 107.0 85.2 98.3 81.4 76.2 76.2 47.9 71.5 44.8 67.7 66.9 65.2 68.2 62.4 47.4 54.4 45.4 52.4 49.1 44.6 47.4 54.4 45.5 47.4 38.0 36.7	91.7 112.9 10 103.0 110.8 11 110.0 110.3 12 80.3 107.8 13 89.0 103.0 14 105.4 98.9 15 90.3 98.0 16 79.5 98.9 17 120.1 98.3 18 88.4 99.0 19 102.1 101.0 20 107.6 101.9 21 109.9 101.9 22 105.5 102.0 23 92.9 101.7 24 114.6 101.6 25 103.5 100.9 26 112.0 97.4 27 83.9 92.2 28 86.7 87.8 29 107.0 85.2 30 98.3 81.4 31 76.2 76.2 32 47.9 71.5 33 44.8 67.7 34 66.9 65.2 35 68.2 62.4 36 47.5 58.4 37 47.4 54.4 38 55.4 52.4 39 49.2 52.0 40 60.8 51.8 41 64.2 51.5 42 46.4 50.4 33 32.0 49.1 44 44.6 47.4 45 38.2 45.5 46 67.8 42.7 47 61.3 39.0 48 28.0 36.7 49 34.3 36.1 50 28.9 34.6 51 29.3 32.6 52 14.6 29.7 53 21.5 25.4 54 33.8 22.4 55 29.1 20.5 56 11.5 19.1 57 23.9 17.8 58	91.7	91.7 112.9 10
112.9 110.8 110.3 107.8 103.0 98.9 98.0 98.9 99.0 101.9 101.9 101.9 101.6 100.9 101.6 100.9 97.4 287.8 85.2 476.5 51.5 42.7 45.5 51.6 51.6 51.6 42.7 36.7 45.7 36.7 47.4 45.7 36.7 36.7 47.8 47.8 47.8 47.8 47.8 47.8 47.8 47	112.9	112.9	112.9 10 -2.1 .36 .06 110.8 11 -0.5 .37 .07 110.3 12 -2.5 .38 .08 107.8 13 -4.8 .38 .08 103.0 14 -4.1 .39 .09 98.9 15 -0.9 .40 .09 98.0 16 +0.9 .40 .10 98.9 17 -0.6 .41 .11 98.3 18 +0.7 .42 .11 99.0 19 +2.0 .43 .12 101.0 20 +0.9 .43 .13 101.9 21 0.0 .44 .13 101.9 22 +0.1 .45 .14 102.0 23 -0.3 .45 .14 102.0 23 -0.3 .45 .14 101.7 24 -0.1 .46 .15 101.7
	10 112 13 145 167 189 190 123 123 123 123 123 133 133 133 133 144 145 146 147 149 159 159 159 159 159 159 159 159 159 15	10	10 -2.1 .36 .06 11 -0.5 .37 .07 12 -2.5 .38 .08 13 -4.8 .38 .08 14 -4.1 .39 .09 15 -0.9 .40 .09 16 +0.9 .40 .10 17 -0.6 .41 .11 18 +0.7 .42 .11 19 +2.0 .43 .12 20 +0.9 .43 .13 21 0.0 .44 .13 22 +0.1 .45 .14 23 -0.3 .45 .14 24 -0.1 .46 .15 24 -0.1 .46 .15 25 -0.7 .47 .16 25 -0.7 .47 .16 25 -0.7 .47 .16 27 -5.2 .48 .17 28 -4.4 .49 .18 29 -2.
-2.1 -0.5 -37 -2.5 -38 -4.8 -39 -4.1 -0.9 -4.0 -0.6 +0.7 +2.0 -4.3 +0.9 -0.1 -0.3 -0.1 -0.7 -3.5 -4.4 -2.6 -3.8 -5.2 -4.4 -2.6 -3.8 -5.2 -4.7 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -4.7 -5.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -5.2 -3.8 -3.9 -6.6 -1.5 -2.0 -6.6 -1.5 -2.0 -6.6 -1.5 -2.0 -6.6 -1.5 -2.0 -6.6 -1.5 -2.0 -6.6 -1.5 -2.0 -3.0 -3.0 -3.0 -3.0 -3.0 -3.0 -3.0 -3	.36 .37 .38 .39 .40 .42 .43 .44 .45 .45 .45 .45 .55 .55 .55 .55 .55		

	101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122	14.6 2.4 12.7 17.7 9.9 14.3 15.0 31.2 2.3 5.1 1.6 15.2 8.8 9.9 14.3 9.9 8.2 24.4 8.7 11.9 15.8 21.6	16.7 16.3 15.1 13.7 12.5 11.7 11.6 11.7 12.0 11.8 11.4 11.7 11.9 10.8 13.1 13.2 12.7 12.9 12.9		-0.4 -1.2 -1.4 -1.2 -0.8 -0.1 +0.1 +0.3 -0.2 -0.4 +0.3 +0.2 -1.1 -0.2 +1.2 +1.3 +0.1 -0.5 +0.2 0.0	.72 .72 .73 .74 .74 .75 .77 .77 .78 .79 .80 .81 .82 .82 .83 .84 .84 .85	.38 .39 .39 .40 .41 .42 .43 .44 .45 .46 .47 .48 .49 .49 .50	
·	123 124 125 126 127 128 129	14.2 6.0 6.3 16.9 6.7 14.2 2.2	12.7 11.5 10.6 10.3 9.5 8.2 7.2	82 83 84 85 86	-1.2 -0.9 -0.3 -0.8 -1.3 -1.0	.87 .88 .89 .89 .90	.51 .52 .53 .53 .54 .54	
	130 131 132 133 134 135 136 137 138	3.3 6.6 7.8 0.1 5.9 6.4 0.1 0.0 5.3	2.3	92 93 94 95 96 97 98	-0.5 -0.7 -0.7 -0.5 -0.2 -0.3 -0.6 -0.1	.92 .93 .94 .95 .96 .96 .97 .98		
DEC 1878	140 141 0 1 2 3 4 5	4.1 0.5 0.5 1.0 0.6 0.0 6.2 2.4	2.5 2.2 2.5 3.2 3.7 4.2 5.1	-58 -57 -56	-0.3 +0.3 +0.7 +0.5 +0.5 +0.9 +0.6		.63 0.63 .63	R _{MIN} SCN 12
	6 7 8	4.8 7.5	5.7 7.0 9.0	-54 -53	+1.3 +2.0 +1.9	•04 •05	.66 .67	

25 26 27 29 31 33 33 34 35 36 37 38 39 41 44 45 46 47 48	6.1 12.3 13.1 24.0 22.3 19.5 19.5 19.5 19.5 19.5 19.5 19.5 19.5	10.9 12.7 15.7 15.7 19.9 17.7 19.9 17.7 19.9 17.7 19.9 17.7 19.9 17.7 19.9 17.7 19.9 17.7 19.9 17.7 19.9 17.7 19.9 19.9	-39 -38 -37 -36 -35 -34 -33 -32 -31 -39 -28 -27 -26 -27 -28 -27 -29 -18 -17 -16 -15 -14 -13 -12	+2.1 +1.9 +2.1 +3.7 +2.1 +1.6 +1.6 +1.6 +1.6 +1.6 +1.7 +2.7 +2.7 +2.1 +2.7 +2.7 +2.7 +2.7 +1.1 +1.0 +1.4 +2.5 +2.7 +2.7 +0.2 -2.0 -2.0 -2.0 -2.0 -2.0 -2.0 -2.7 +1.1 +1.0 +1.1 +1.0 +1.1 +1.0 +1.1 +1.0 +1.0	.07 .08 .09 .10 .11 .12 .13 .14 .15 .16 .17 .18 .19 .21 .22 .23 .24 .25 .25 .27 .28 .29 .31 .33 .34 .35 .36 .37	.69 .69 .70 .71 .72 .73 .74 .74 .75 .76 .76 .77 .78 .79 .79 .80 .81 .81 .82 .83 .83 .84 .84 .85 .86 .86 .87 .88 .89 .90 .91 .91	ĠPV ^R 13
44 45 46 47	40.4 57.7 59.2 84.4	60.0 58.1 56.5 54.5	-16 -15 -14 -13	-1.9 -1.6 -2.0 0.0	.33 .34 .34 .35	.90 .91 .91 .92	
49 50 51 52 53	60.6 46.9 42.8 82.1 31.5 76.3 80.6 46.0 52.6 83.8 84.5	57.2 58.9 58.9 59.8 60.8 62.2 64.9 67.9 71.4 73.6 74.2	-11 -10 - 9 - 8 - 7 - 6 - 5 - 4 - 3 - 2 - 1	+1.7 0.0 +0.9 +1.0 +1.4 +2.7 +3.0 +3.5 +2.2 +0.6 +0.4	.37 .37 .38 .39 .40 .40 .41 .42 .43 .43		

<u>.</u>	60 61 62 63 64 65 66 67 77 77 77 77 77 77 77 77 77 77 77	75.9 91.5 86.9 86.8 76.1 66.5 51.2 53.1 55.8 61.9 47.8 61.9 47.8 71.8 73.0 73.0 73.0 73.7 730.3 730.7 727.1 730.3	74.6 72.4 71.6 72.4 71.3 67.8 64.6 61.4 58.6 54.2 53.6 55.2 57.1 56.2 54.3 51.4 47.2 45.0 47.4 47.2 45.0 37.0 32.0 30.0 27.4 25.8 26.8 13.9 13.0 12.7 12.7 12.7	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 12 21 22 23 24 25 26 27 28 29 30 31 31 31 31 31 31 31 31 31 31 31 31 31	-2.2 -0.8 +0.8 -1.15 -3.2 -3.2 -3.6 -2.4 -0.6 -1.3 -1.6 -1.3 -1.6 -1.3 -1.6 -1.3 -1.6 -1.3 -1.6 -1.3 -1.6 -1.2 -1.3 -1.6 -1.3 -1.6 -1.3 -1.6	•46 •47 •48 •49 •51 •51 •52 •53 •54 •55 •55 •57 •58 •60 •61 •63 •63 •64 •65 •67 •67 •67 •67 •67 •67 •67 •67 •67 •67	0.00 .01 .02 .03 .04 .05 .07 .08 .09 .10 .11 .12 .13 .14 .15 .16 .17 .17 .18 .19 .20 .21 .22 .23 .24 .25 .26 .27 .28 .29 .30 .31 .31 .32 .33 .34 .35 .36 .37 .38 .39 .30 .30 .30 .30 .30 .30 .30 .30	R _{MAX} AND
	97 98 99 100 101	10.3 13.2 4.2 6.9 20.0	13.1 13.0 12.6 11.9 12.1	37 38 39 40 41	-0.1 -0.4 -0.7 +0.2 +0.6	.72 .73 .74 .75	.31 .31 .32 .33	

	111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134	7.8 5.1 7.0 7.1 3.1 2.8 8.8 2.1 10.7 6.7 0.8 8.5 6.7 4.3 2.4 6.4 9.4 20.6 6.5 2.1 0.2 6.7 5.3 0.6	7.9 7.8 7.3 6.3 5.8 5.5 5.5 5.7 6.2 7.1 6.2 6.3 5.9 5.7 5.6 5.9 5.0	51 52 53 54 55 56 57 59 61 62 63 64 66 67 68 69 71 72 73	-0.1 0.0 -0.5 -1.0 -0.5 0.0 0.0 -0.3 -0.2 +0.2 +1.0 +0.7 -0.1 -0.4 -0.5 +0.2 -0.1 -0.4 -0.2 -0.4 -0.2 -0.4 -0.2	.83 .84 .84 .85 .86 .87 .88 .89 .90 .91 .92 .93 .93 .94 .95 .96 .96 .97 .98 .99 .99	.42 .43 .44 .45 .46 .47 .48 .49 .50 .51 .52 .53 .54 .55 .55 .56 .57 .58 .60 .61	
FEB 1890	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25	0.6 5.1 1.6 4.8 1.3 11.6 8.5 17.2 11.2 9.6 7.8 13.5 22.2 10.4 20.5 41.1 48.8 33.2 53.8 51.5 41.9 369.1 75.6 49.9	5.0 5.0 5.8 6.6 7.4 8.6 9.8 10.8 13.1 16.5 20.5 23.5 26.0 29.2 34.6 37.9 42.5 46.3 50.0 53.7 56.5 58.4 62.0 65.2	-47 -46 -45 -44 -43 -42 -41 -40 -38 -37 -36 -35 -34 -32 -31 -29 -28 -27 -26 -24 -23 -22	0.0 +0.8 +0.4 +0.4 +1.2 +1.0 +2.3 +3.0 +2.5 +3.0 +2.5 +3.0 +2.5 +3.7 +3.8 +3.7 +3.8 +3.7 +3.8 +3.7 +3.2 +1.2	0.00 .01 .01 .02 .03 .03 .04 .05 .06 .07 .08 .09 .10 .11 .12 .13 .13 .14 .15 .15 .16 .17	0.61 .62 .63 .64 .65 .66 .67 .68 .69 .70 .71 .72 .73 .74 .75 .76 .77 .78 .79 .80 .81 .82	R̄ _{MIN} SCN 13

JAN 1894	26 69.6 27 79.6 28 76.3 29 76.8 30 101.4 31 62.8 32 70.5 33 65.4 35 75.0 36 73.0 37 65.7 38 88.1 39 84.7 40 89.9 41 129.2 43 77.2 44 83.2 45 75.1 46 83.2 47 84.6 47 84.6 48 47 84.6 48 47 84.6 48 47 85.3 50 81.6 51 101.2 52 106.0 54 70.3 55 66.6 57 56.6 68 67.5 58 60.0 59 67.5 58 60.0 59 67.5 58 60.0 59 67.5 58 60.0 59 67.5 58 67.7 58 68.9 57.7 58 68.9 57.7 58 68.9 57.7 58 68.9 57.7 58 68.9 57.7 58 68.9 57.7 58 68.9 57.7 58 69 77.7 71 72 9.0 72 77.7 7	66.4 71.0 73.4 73.9 75.3 77.7 78.8 81.6 83.4 85.8 86.2 86.3 87.7 75.4 88.8	-21 -20 -18 -16 -17 -18 -17 -18 -17 -18 -17 -18 -18 -18 -19 -18 -19 -19 -19 -19 -19 -19 -19 -19 -19 -19	+1.7 +2.2 +0.5 +1.0 +1.0 +1.0 +1.0 +1.0 +1.0 +1.0 +1.0	.18 .19 .20 .21 .22 .23 .24 .25 .27 .28 .29 .20 .31 .32 .33 .34 .35 .36 .37 .38 .39 .41 .42 .43 .44 .45 .45 .46 .47 .48 .48 .49 .49 .49 .49 .49 .49 .49 .49 .49 .49	.83 .83 .84 .85 .86 .87 .88 .89 .91 .92 .93 .94 .95 .97 .98 .99 .97 .98 .99 .90 .01 .02 .03 .03 .04 .05 .06 .07 .08 .09 .10 .11 .12 .13 .14 .14 .15 .17 .17 .19 .19 .19 .19 .19 .10 .10 .10 .10 .10 .10 .10 .10 .10 .10	ĀMAX
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	128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143	12.9 4.5 0.3 0.2 2.4 4.5 0.0 10.2 5.8 0.7 1.0 0.6 3.7 3.8 0.0 5.5	6.8 5.9 5.4 4.8 4.4 3.9 3.2 2.8 3.0 3.1 3.3 3.6 3.3 2.8 2.7	81 82 83 84 85 86 87 88 90 91 92 93 94 95	-0.9 -0.5 -0.6 -0.4 -0.5 -0.7 -0.4 0.0 +0.2 +0.1 +0.2 +0.3 -0.3 -0.5 -0.1 0.0	.90 .90 .91 .92 .92 .93 .94 .94 .95 .96 .97 .97 .98 .99	.56 .57 .58 .59 .60 .61 .62 .63 .63 .64 .65	
JAN 1902	0 1 2 3 4 5 6 7 8 9 0 11 12 13 14 15 16 7 18 19 0 12 22 23 24 25 26 27 28 29 30 31 32 32 32 32 32 32 32 32 32 32 32 32 32	5.5 0.0 12.4 0.0 2.8 1.4 0.9 2.6 16.3 1.1 8.3 17.0 13.5 14.6 16.3 27.9 28.8 11.1 38.5 45.6 31.6 31.6 31.7 45.6 31.6 31.7 45.6 31.6 31.7 45.6 31.6 31.7 45.6 31.7 45.6 31.6 31.7 45.6 45.7 45.	2.7 2.7 3.1 3.9 5.0 6.7 9.5 10.3 14.5 15.9 19.3 15.9 22.4 26.9 29.6 27.7 41.5 42.9 46.4 49.8 50.4	-49 -48 -47 -46 -45 -44 -43 -41 -40 -38 -37 -36 -37 -38 -37 -39 -27 -26 -27 -29 -18 -17 -16	0.0 +0.4 +0.8 +0.3 +0.2 +0.8 +0.7 +1.6 +1.1 +1.7 +2.2 +1.3 +1.1 +2.4 +3.2 +1.3 +1.7 +1.8 +2.1 +2.0 +2.2 +1.4 +0.4 +0.1 +1.3 +0.4 +0.5 +0.6 +0.6 +0.6 +0.6 +0.6 +0.6 +0.6 +0.6	0.00 .01 .01 .02 .03 .04 .04 .05 .06 .07 .08 .09 .10 .11 .12 .13 .14 .14 .15 .16 .17 .17 .18 .19 .20 .21 .22 .23 .24	.66 .67 .68 .68 .69 .70 .71 .72 .73 .74 .74 .75 .76 .77 .77 .78 .79 .80 .81 .82 .83 .83 .84 .85 .86 .87 .88 .88 .88	R _{MIN} SCN 14

	34 38.0 35 54.6 36 54.8 37 85.8 38 56.5 39 39.3 40 48.0 41 49.0 42 73.0 43 58.8 44 55.0 45 78.7 46 107.2	50.6 51.3 52.5 53.5 54.5 56.6 60.5 63.4 63.1 60.4 58.5 59.5	-15 -14 -13 -12 -11 -10 - 9 - 8 - 7 - 6 - 5 - 4 - 3	+0.7 +1.2 +1.0 +1.0 +2.1 +3.9 +2.9 -0.3 -2.7 -1.9 +1.0 +1.1	.25 .25 .26 .27 .28 .29 .30 .30 .31 .32 .33	.90 .90 .91 .92 .93 .94 .94 .95 .97 .97	∆ GPV ^R 13
	47 55.5 48 45.5	61.5 63.4	- 2 - 1	+1.9 +0.8	.34 .35 .36	•99	_
FEB 1906	49 31.3	64.2	0	-0.4	.36	0.00	\overline{R}_{MAX}
	50 64.5 51 55.3	63.8 61.3	1 2 3 4 5 6	-2.5 -5.4	.36 .37 .38	.01 .01	*
	52 57.7	55.9	3	-5.4 -2.4	.38	.02	
	53 63.2	53.5	4	+1.6	•38	.03	
	54 103.3 55 47.7	55.1 59.6	5 6	+4.5 +3.1	•39 •40	•04 •04	
	56 56.1	62.7	7	-0.3	.41	• 05	
	57 17.8	62.4	8	-0.7	.41	.06	
	58 38.9	61.7	9	-1.6	•42	•07	
	59 64.7 60 76.4	60 . 1 56 . 9	10 11	-3.2 -1.9	.43 .43	.07 .08	
	61 108.2	55.0	12	+1.4	.44	•09	
	62 60.7	56.4	13	+3.2	•45	•09	
	63 52.6	59.6	14	+2.9	•46	.10	
	64 43.0 65 40.4	62 . 5 62 . 8	15 16	+0.3 -2.3	•46 •47	.11 .12	
	66 49.7	60.5	17	-4.7	.48	.12	
	67 54.3	55.8	18	-4.4	• 49	.13	
	68 85.0	51.4	19	-1.1	.49	•14	
	69 65.4 70 61.5	50.3 50.4	20 21	+0.1 +0.2	.50 .51	.14 .15	
	71 47.3	50.6	22	-0.1	.51	.16	
	72 39.2	50.5	23	+1.1	•52	•17	
	73 33.9	51.6	24	+1.6	•53	.17 .18	
	74 28.7 75 57.6	53.2 51.9	25 26	-1.3 -2.0	• 54 • 54	.19	
	76 40.8	49.9	27	-1.0	• 55	.20	
	77 48.1	48.9	28	+0.4	•56	•20	
	78 39.5 79 90.5	49.3 50.5	29 30	+1.2 +2.1	•57 •57	•21 •22	
	80 86.9	52.6	31	+0.5	•58	•22	
	81 32.3	53.1	32	-1.2	• 59	.23	
	82 45.5 83 39.5	51.9 50.6	33 34	-1.3 -1.2	•59 •60	•24 •25	
	84 56.7	49.4	35	-3.0	.61	.25	

85	46.6	46.4	36	-4.8	.62	.26	∆GNV ^R 13
86	66.3	41.6	37	-0.9	.62	.27	
87	32.3	40.7	38	+1.5	.63	.28	
88	36.0	42.2	39	+1.1	•64	.28	
89	22.6	43.3	40	-0.7	•64	.29	
90	35.8	42.6	41	-1.9	•65	.30	
91	23.1	40.7	42	-2.5	.66	.30	
92	38.8	38.2	43	-2.8	.67	.31	
93	58.4	35.4	44	-1.6	•67	•32	
94	55.8	33.8	45	-1.0	•68	•33	
95	54.2	32.8	46	-1.3	•69	.33	
96	26.4	31.5	47	-1.4	•70	.34	
97	31.5	30.1	48	-1.0	• 70	.35	
98	21.4	29.1	49	-1.4	• 71	.36	
99	8.4	27.7	50	-3.0	• 72	•36	
100	22.2	24.7	51	-4.1	• 72	•37	
101	12.3	20.6	52	-3.0	• 73	.38	
102	14.1	17.6	53	-1.9	• 74	.38	
103	11.5	15.7	54	-1.5	• 75	.39	
104	26.2	14.2	55	-0.2	• 75	.40	
105	38.3	14.0	56	-0.2	• 76	.41	
106	4.9 5.8	13.8 12.8	57 58	-1.0 -0.8	.77 .78	.41 .42	
	3.4 9.0	12.0 11.2	59 60	-0.8 -1.2	• 78 • 79	•43 •43	
110	7.8	10.0	61	-2.5	. 80	•44	
111	16.5	7.5	62	-1.5	. 80	•45	
112	9.0	6.0	63	-0.2	• 81	•46	
113	2.2	5.8	64	-0.2	• 82	•46	
114	3.5	5.6	65	-0.5	• 83	•47	
115	4.0	5.1	66	-0.5	• 83	•48	
116	4.0	4.6	67	-0.6	• 84	•49	
117	2.6	4.0	68	-0.7	• 85	•49	
118	4.2	3.3	69	-0.2	• 86	•50	
119 120	2.2	3.1 3.2	70 71	+0.1 -0.2	. 86 . 87	.51 .51	
121	0.0	3.0	72	+0.1	• 88	.52	
122	4.9	3.1	73	+0.3	• 88	.53	
123 124	4.5 4.4	3.4 3.4	74 75	0.0	• 89 • 90	•54 •54	
125	4.1	3.4	76	+0.3	•91	•55	
126	3.0	3.7	77	+0.2	•91	•56	
127	0.3	3.9	78	-0.1	•92	•57	
128	9.5	3.8	79	-0.3	.93	•57	
129	4.6	3.5	80	-0.3	.93	•58	
130	1.1	3.2	81	-0.4	•94	•59	
131	6.4	2.8	82	-0.2	•95	•59	
132	2.3	2.6	83	-0.1	•96	.60	
133	2.9	2.5	84	-0.3	•96	.61	
134	0.5	2.2	85	-0.4	•97	.62	
135	0.9	1.8	86	-0.1	•98	.62	

	136 137 138	0.0 0.0 1.7	1.7 1.6 1.5	87 88 89	-0.1 -0.1 0.0	.99 .99 1.00	.63 .64 .64	
JUL 1913	012345678901121141617890122222223312334567890123444444444444444444444444444444444444	1.0.2.2.1.7.8.8.6.1.3.2.4.4.7.7.2.4.3.0.8.6.6.5.5.5.3.4.0.8.5.7.5.2.1.7.6.0.7.9.8.4.5.0.7.0.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2.2	1.5.5.6.4.3.9.6.0.8.5.4.8.4.9.1.6.7.3.4.8.9.3.3.4.5.5.6.4.8.4.9.1.6.7.3.4.8.9.3.3.4.5.5.6.4.8.4.9.1.6.7.3.4.8.9.3.3.4.5.5.5.5.5.5.5.5.5.5.5.5.5.5.5.5.5		0.0 +0.8 +0.6 +0.7 +0.4 +0.7 +0.4 +0.7 +0.4 +1.5 +1.6 +1.5 +1.7 +1.8 +1.6 +1.7 +1.8 +1.6 +1.8 +1.8 +1.8 +1.8 +1.8 +1.8 +1.8 +1.8	0.00 .01 .02 .03 .03 .04 .05 .06 .07 .08 .09 .10 .11 .13 .13 .14 .15 .16 .17 .18 .19 .20 .21 .22 .23 .24 .25 .26 .27 .28 .29 .30 .31 .32 .33 .33 .33 .33 .33 .33 .33 .33 .33	.64 .65 .66 .67 .68 .69 .70 .71 .72 .73 .74 .75 .75 .76 .77 .78 .80 .81 .82 .83 .84 .85 .86 .87 .88 .89 .91 .91 .91 .92 .93 .94 .95 .96 .96 .96 .96 .96 .96 .96 .96 .96 .96	\bar{R}_{MIN} SCN 15
	45 46	74.7 114.1	94.1 96.3	- 4 - 3	+2.2 +4.4	.38 .38	•97 •98	

AUG 1917	47 114.9 48 119.8 49 154.5 50 129.4 51 72.2 52 96.4 53 129.3 54 96.0 55 65.3 56 72.2 57 80.5 58 76.7 59 60 107.6 61 101.7 62 85.0 64 83.4 65 59.2 66 48.1 77 73 69.0 77 75 52.8 70 88 111.2 72 64.7 73 69.0 74 54.7 75 52.8 81 129.3 82 33.3 83 38.7 84 27.5 85 36.3 87 49.6 88 29.9 90 31.5 91 28.3 92 95 32.4 93 32.4 94 97 22.8	100.7 104.8 105.4 104.2 103.5 102.2 98.3 95.5 92.8 88.5 77.0 83.5 78.6 77.2 77.5 76.1 75.4 78.0 78.4 75.2 72.8 60.5 56.7 61.9 60.5 56.7 51.4 46.8 43.2 40.3 39.4 39.4 39.4 39.6 31.0 31.7 31.1 29.0 27.3 24.4	 210123456789012345678901232222222333333333442345678 444444444444444444444444444444444444	+4.1 +0.2 -1.3 -2.7 -1.3 -2.7 -1.3 -2.7 -1.3 -2.7 -1.3 -2.4 -1.3 -1.3 -1.3 -1.3 -1.3 -1.3 -1.3 -1.3	.39 0.41 0.43 0.445 0.447 0.4489 0.4423 0.445 0.449 0.	.99 .99 .001 .002 .003 .005 .007 .009 .011 .113 .114 .115 .116 .117 .119 .123 .123 .124 .125 .126 .127 .129 .133 .134 .136 .137 .137 .137 .137 .137 .137 .137 .137	R _{MAX} R ₁₃ .
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	98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120	17.8 18.2 17.8 20.3 11.8 26.4 54.7 11.0 8.0 5.8 10.9 6.5 4.7 6.2 7.4 17.5 4.5 1.5 3.3 6.1 3.2 9.1 3.5	25.5 25.8 24.3 22.5 20.1 18.1 16.9 15.8 14.9 14.4 13.9 14.4 7.1 6.6 6.4 5.0 6.6 6.9 6.4 5.6	49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 70 71	+0.3 -1.5 -1.8 -2.4 -2.0 -1.2 -1.1 -0.9 -0.5 -0.5 -1.3 -3.2 -2.3 -0.4 -0.1 -0.2 -0.5 +0.6 +0.3 -0.5 -0.8 0.0	.82 .83 .83 .84 .85 .86 .87 .88 .89 .90 .91 .92 .93 .93 .94 .95 .96 .97 .98 .99	.38 .39 .40 .41 .41 .42 .43 .44 .45 .45 .45 .46 .47 .48 .49 .50 .51 .52 .52 .53 .54	
JUL 1923	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 26 27 27 28 29 29 20 20 20 20 20 20 20 20 20 20 20 20 20	3.5 0.5 13.2 11.6 10.0 2.8 0.5 5.1 1.8 11.3 20.8 24.0 28.1 19.3 25.6 22.5 16.5 5.5 23.2 18.0 31.7 42.8 47.5 38.5 37.9 60.2	5.6 5.7 5.8 6.8 8.1 9.8 11.6 12.9 14.0 15.1 16.9 17.9 19.3 20.8 22.6 24.5 25.9 27.1 29.4 32.6 36.0 40.9 47.1 51.8 55.6	-57 -56 -55 -54 -53 -52 -51 -50 -48 -47 -46 -45 -44 -43 -41 -40 -39 -38 -36 -34 -32 -31	0.0 +0.1 +0.1 +1.0 +1.3 +1.7 +1.8 +1.3 +1.1 +1.1 +1.0 +0.8 +1.0 +1.4 +1.5 +1.8 +1.9 +1.4 +1.2 +2.3 +3.2 +4.9 +6.2 +4.7 +3.8 +2.1	0.00 .01 .02 .02 .03 .04 .05 .06 .07 .07 .08 .09 .10 .11 .11 .12 .13 .14 .15 .16 .16 .17 .18 .19 .20 .21	.55 .56 .57 .58 .59 .60 .61 .62 .63 .63 .64 .65 .66 .67 .68 .69 .70 .70 .71 .72 .73 .73 .74 .75 .76	R̄ _{MIN} SCN 16

APR 1928	27 69.2 28 58.6 29 98.6 30 71.8 31 70.0 32 62.5 33 38.5 34 64.3 35 73.5 36 60.8 37 61.6 81.6 41 79.4 42 43 93.0 44 69.6 45 79.1 47 48 54.9 49 53.8 50 68.4 51 63.1 52 53 45.2 53 85.4 64 59.6 67 68.9 68 69 61.4 68 69 61.4 69 66 68.9 60 88.8 61 68 69 70 71.7 65 68 69 70 71.7 70 65 81.1 71 72 73 65 81.1 73 74 75 76 76 77 70 70 70 70 70 70 70 70 70 70 70 70	57.7 50.9 62.5 64.1 65.2 64.3 77.7 71.7	- 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 0 1 1 2 1 3 1 4 1 5 6 1 7 8 1 9	+2.0 +1.6 +1.0 +1.0 +1.2 -0.7 -0.4 +1.2 +2.6 -0.5 -0.0 -0.1 -1.4 -0.7 -0.2 -0.7 -1.3 -1.6 -1.9 -1.9 -1.9 -1.9 -1.9 -1.9 -1.9 -1.9	223455678900123 3344567899001123 3344567899001123 344567899001123	.77 .78 .79 .80 .81 .82 .83 .84 .85 .86 .87 .88 .89 .91 .92 .93 .94 .95 .96 .97 .98 .99 .91 .92 .93 .94 .95 .96 .06 .06 .07 .09 .09 .00 .00 .00 .00 .00 .00 .00 .00	R _{MAX}
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	78 79 80 81 82 83 84 85 86 87 88 90 91 92 93 94 95 96 97 99 101 102 103 104 110 110 111 112 113 114 115 116 117 118 119 119 119 119 119 119 119 119 119	65.3 49.9 35.0 38.8 21.9 24.9 32.1 35.6 43.0 24.6 13.0 10.0 11.2 11.2 9.8 8.9 21.9 22.1 12.1 29.8 8.9 21.9 22.1 22.1 22.1 22.1 22.1 23.1 24.9 25.1 26.0 26.0 26.0 26.0 26.0 26.0 26.0 26.0	53.6 49.9 48.1 47.5 33.6 23.6 22.6 23.6 22.6 21.7 17.5 14.8 14.2 11.6 11.7 7.6 4.9 6.1 3.5 11.7 11.7 11.7 11.7 11.7 11.7 11.7 11	2123456789012345678901234567890123456789012345666666666666666666666666666666666666	-3.7 -1.8 -0.9 -2.7 -5.3 -5.6 -2.4 -0.5 -0.5 -0.8 -1.0 -0.7 -1.6 -1.7 -2.0 -1.1 -0.0 -0.7 -0.5 -0.7 -0.7 -0.9 -0.7 -0.9 -0.7 -0.9 -0.7 -0.9	.75 .76 .77 .78 .79 .80 .81 .82 .83 .84 .85 .86 .87 .88 .89 .90	.19 .20 .21 .22 .23 .24 .25 .26 .27 .28 .29 .31 .32 .33 .34 .35 .37 .38 .39 .40 .41 .42 .43 .44 .45 .47 .48 .49 .51 .55 .56 .56 .56 .56 .56 .56 .56 .56 .56	Δ̄R̄13
SEP 1933	0 1 2 3 4	5.1 3.0 0.6 0.3 3.4	3.5 3.6 4.6 5.4 5.7	-43 -42 -41 -40 -39	+0.1 +1.0 +0.8 +0.3 +0.6	0.00 .01 .02 .02 .03	.60 .61 .62 .63	R _{MIN} SCN 17

56 7 8 9 9 0 1 2 3 4 5 6 6 7 8 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9	83.3 60.4 83.5 67.5 105.5 66.5 65.4 46.8 45.6 46.8	107.2 109.4 108.8 106.3 103.6	13 14 15 16 17 18 19 20 21 22 22 22 23 33 33 33 33 33 44 44 45 47 48 49 50 51 51 51 51 51 51 51 51 51 51 51 51 51	+2.6.5.7.5.1.2.7.2.5.5.7.3.4.0.0.2.7.3.0.3.9.3.8.7.9.8.2.7.2.0.1.9.9.4.2.3.8.1.7.4.0.2.0.8.6.2.1.9.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0	.45 .46 .47 .49 .50 .51 .53 .54 .55 .57 .58 .59 .61 .62 .63 .64 .65 .66 .67 .70 .71 .74 .75 .77 .78 .78 .79 .81 .82 .82	.11 .12 .13 .14 .15 .16 .17 .18 .19 .20 .21 .22 .23 .24 .25 .26 .27 .28 .29 .30 .31 .32 .33 .34 .35 .36 .37 .38 .39 .40 .40 .42 .43 .44 .45 .45 .45 .45 .45 .45 .45 .45 .45	\triangle_{GNV}^{R} 13
101	52.8	40.1	58	-3.6	• 81 • 82	.48 .49	
105 106	11.4 17.7	31.1 29.6	62 63	-1.5 -1.9	• 84 • 85	•51 •52	

	107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125	20.2 17.2 19.2 30.7 22.5 12.4 28.9 27.4 26.1 14.1 7.6 13.2 19.4 10.0 7.8 10.2 18.8 3.7 0.5	27.7 25.6 23.0 21.1 20.5 20.1 19.9 19.6 18.8 17.5 16.5 16.0 14.4 12.6 10.8 9.2 8.6 8.2 7.7	64 65 66 67 68 69 70 71 72 73 74 75 76 77 80 81 82	-2.1 -2.6 -1.9 -0.6 -0.4 -0.2 -0.3 -0.8 -1.3 -1.0 -0.5 -1.6 -1.8 -1.6 -0.6 -0.4 -0.5 +0.1	.86 .87 .88 .89 .90 .91 .92 .93 .94 .95 .96 .97 .98 .98	.53 .54 .55 .55 .56 .57 .58 .59 .60 .61 .62 .63 .64 .64 .65	
FEB 1944	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 22 22 22 23 23 23 23 23 23 23 23 23	0.5 11.0 0.3 2.5 5.0 16.7 14.3 16.9 10.8 28.4 18.5 12.7 21.5 32.6 36.2 42.6 25.9 34.9 68.8 46.0 27.4 47.6 86.2 76.6 75.7 84.9 73.5 116.2 116.2	7.7 7.8 8.4 8.8 9.2 10.2 11.3 12.3 14.0 16.5 19.0 21.9 23.8 25.1 28.1 31.7 33.1 34.3 38.6 43.9 48.1 52.1 56.0 60.6 67.0 72.9 76.8 81.4 88.6 95.3 100.2	-39 -38 -37 -36 -37 -35 -31 -32 -31 -28 -27 -28 -27 -28 -27 -29 -18 -17 -13 -11 -10 -19	+0.6 +0.4 +1.0 +1.7 +2.5 +2.9 +1.3 +3.6 +1.2 +3.6 +1.2 +4.3 +4.0 +4.6 +5.9 +4.6 +7.2 +4.1 +4.1	0.00 .01 .02 .02 .03 .04 .05 .06 .07 .07 .08 .09 .10 .11 .11 .12 .13 .14 .15 .16 .16 .17 .18 .19 .20 .21 .22 .23 .24 .25	.68 .69 .69 .70 .71 .72 .73 .74 .75 .76 .77 .78 .79 .79 .80 .81 .82 .83 .83 .84 .85 .86 .87 .88 .89 .90 .91 .92	R _{MIN} SCN 18

	31 94.4 32 102.3 33 123.8 34 121.7 35 115.7 36 133.4 37 129.8	104.3 109.6 117.6 126.2 131.7 136.8 143.4	- 7 - 6 - 5 - 4 - 3 - 2	+5.3 +8.0 +8.6 +5.5 +5.1 +6.6 +5.6	.25 .26 .27 .28 .29 .30	.93 .94 .95 .96 .97	△ GPV ^R 13
MAY 1947	38 149.8 39 201.3 40 163.9 41 157.9 42 188.8 43 169.4 44 163.6 45 128.0 46 116.5 47 108.5 48 86.1 49 94.8 50 189.7 51 174.0 52 167.8 53 142.2 54 157.9 55 143.3 56 136.3 57 95.8 58 138.0 59 119.1 60 182.3 61 157.5 62 147.0 63 106.2 64 121.7 65 125.8 66 123.8 67 145.3 68 131.6	149.0 151.8 151.7 151.2 148.9 145.5 145.7 146.2 145.3 144.8 142.8 140.5 138.2 135.8 135.3 136.6 141.1 147.7 148.5 143.9 139.2 136.6 134.5 133.2 134.8 136.0 134.4 130.0 124.4 121.0	4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28	+2.8 -0.5 -2.3 -3.4 +0.5 -0.5 -0.5 -0.5 -2.3 -2.3 -2.4 -0.5 +1.5 +6.6 +0.8 -4.6 -4.7 -2.6 -1.3 -1.8 +1.6 -4.4 -3.4 -1.4	.31 .333 .344 .35 .378 .399 .412 .433 .445 .447 .488 .490 .512 .523 .545 .560 .556	.99 0.00 .01 .02 .02 .03 .04 .05 .06 .07 .08 .09 .10 .11 .12 .12 .13 .14 .15 .15 .16 .17 .18 .19 .20 .21 .22	R _{MAX}
	68 131.6 69 143.5 70 117.6 71 101.6 72 94.8 73 109.7 74 113.4 75 106.2 76 83.6 77 91.0 78 85.2 79 51.3 80 61.4 81 54.8	121.0 119.6 118.0 115.0 111.9 106.4 99.5 92.9 86.6 82.2 79.0 75.3 72.2 71.4	29 30 31 32 33 34 35 36 37 38 39 40 41 42	-1.4 -1.6 -3.0 -3.1 -5.5 -6.9 -6.6 -6.3 -4.4 -3.2 -3.7 -3.1 -0.8 +0.9	.56 .57 .58 .59 .60 .61 .62 .63 .64 .65 .66	.22 .23 .24 .25 .25 .26 .27 .28 .28 .29 .30 .31 .32	[∆] GNV ^R 13

	82 83 84 85 86 87 88 99 99 99 99 101 103 104 105 107 109 111 113 114 115 117 118 119 112 112 112 112 112	54.1 59.9 59.9 92.9 100.5 61.0	72.3 71.7 69.5 69.8 70.2 68.6 63.3 59.2 46.8 43.2 53.0 46.3 30.8 42.0 31.9 30.8 42.7.1 24.1 21.6 19.9 17.4 10.4 8.6 6.3 5.2 11.4 10.4 1	53 54 55 56 57 58 59 61 62 63 64 65 67 71 73 74 75 79 80	-0.6 -2.2 +0.3 +0.9 -0.4 -1.2 -2.3 -3.0 -4.1 -6.2 -3.6 -1.2 -3.4 -2.5 -1.7 -1.1 -1.2 -0.6 -0.5 -1.7 -1.0 -1.5 -2.4 -1.3 -0.1 -1.6 -1.4 -1.2 -0.8 +0.3	.98 .99 .99	.33 .34 .35 .35 .36 .37 .38 .39 .41 .42 .43 .44 .45 .45 .47 .48 .49 .51 .52 .53 .55 .55 .55 .55 .55 .55 .55 .55 .55	
APR 1954	0 1 2 3 4 5 6 7 8	1.8 0.8 0.2 4.8 8.4 1.5 7.0 9.2 7.6	3.4 3.7 4.2 5.4 7.2 7.8 7.9 9.5	-47 -46 -45 -44 -43 -42 -41 -40 -39	+0.3 +0.5 +1.2 +1.8 +0.6 +0.1 +1.6 +2.5 +2.2	0.00 .01 .02 .02 .03 .04 .05 .06	.64 .65 .65 .66 .67 .68 .68	R _{MIN} SCN 19

	9 23.1 10 20.8 11 4.9 12 11.3 13 28.9 14 31.7 15 26.7 16 40.7 17 42.7 18 58.5 19 89.2 20 76.9 21 73.6 22 124.0 23 118.4 24 110.7 25 136.6 26 116.6 27 129.1 28 169.6 29 173.2 30 155.3 31 201.3 32 192.1 33 165.0 34 130.2 35 157.4	14.2 16.4 19.5 23.4 28.8 35.1 40.1 46.5 55.5 64.4 73.0 81.0 88.8 98.5 109.3 118.7 127.4 136.9 145.5 149.6 151.5 155.8 159.6 164.3 170.2 172.2 174.3	-38 -37 -36 -35 -34 -33 -32 -31 -30 -29 -28 -27 -26 -25 -24 -22 -21 -20 -19 -18 -15 -14 -13 -12	+2.2 +3.1 +3.9 +5.4 +6.3 +5.0 +6.4 +9.9 +8.6 +7.8 +9.7 +10.8 +9.7 +10.8 +9.5 +4.1 +1.9 +4.3 +4.7 +5.9 +2.0 +2.1 +6.7	.07 .08 .09 .10 .11 .12 .13 .13 .14 .15 .16 .17 .17 .18 .19 .20 .21 .21 .22 .23 .24 .25 .25 .26 .27 .28	.71 .72 .73 .74 .75 .75 .76 .77 .78 .79 .80 .81 .82 .82 .83 .84 .85 .85 .86 .87 .88 .89	ĠPV ^R 13
MAR 1958	36 175.2 37 164.6 38 200.7 39 187.2 40 158.0 41 235.8 42 253.8 43 210.9 44 239.4 45 202.5 46 164.9 47 190.7 48 196.0 49 175.3 50 171.5 51 191.4 52 200.2 53 201.2 54 181.5 55 152.3 56 187.6 57 217.4 58 143.1 59 185.7	181.0 185.5 187.9 191.4 194.4 197.3 199.5 200.8 200.0 199.0 201.3 196.8 191.4 186.8 185.2 184.9 183.8 182.2 180.7 180.5 178.6 176.9 174.5	-11 -10 - 9 - 8 - 7 - 6 - 5 - 4 - 3 - 2 - 1 0 1 2 3 4 5 6 7 8 9 10 11 12	+4.5 +2.4 +3.5 +3.0 +2.9 +2.2 +1.3 -0.8 -1.0 +1.9 +0.4 -4.5 -5.4 -4.6 -1.6 -1.5 -0.3 -1.1 -1.5 -0.2 -1.9 -1.7 -2.4 -5.3	.29 .29 .30 .31 .32 .33 .34 .35 .36 .37 .38 .39 .40 .41 .42 .43 .44 .45 .46 .47	.92 .93 .94 .95 .95 .96 .97 .98 .99 0.00 .01 .02 .03 .04 .05 .06 .07 .08	R̄ _{MA} χ

$\begin{array}{c} 601\\ 612\\ 345\\ 667\\ 777\\ 777\\ 777\\ 777\\ 777\\ 777\\ 77$	163.3 172.0 168.7 149.6 199.6 145.2 111.4 106.0 102.0 119.6 110.2 121.0 110.2 121.0 110.2 121.0	169.2 165.1 161.4 155.8 151.3 141.1 137.1 132.5 128.9 125.6 119.6 117.0 113.9 102.4 97.3 87.9 93.3 87.9 93.3 87.9 88.7 89.4 89.4 89.4 89.4 89.7 89.8 89.7 89.8 89.7 89.8	134567890123456789012333333334445678901234567890123	-4.1765020669406132564254962376299854042959824416455 -4.3.3.4.2.5.4.9.6.2.3.76299824416455 -4.3.4.2.6.4.2.5.4.9.6.2.3.76299824416455	.48 .49 .51 .52 .53 .54 .55 .56 .67 .63 .64 .65 .67 .71 .72 .73 .74 .75 .77 .77 .77 .77 .77 .77 .77 .77 .77	.10 .11 .12 .13 .14 .15 .16 .17 .18 .19 .20 .21 .22 .23 .24 .25 .27 .28 .29 .30 .31 .32 .33 .34 .35 .36 .37 .38 .39 .40 .41 .42 .43 .44 .45 .46 .46 .46 .47 .48 .48 .48 .48 .48 .48 .48 .48 .48 .48	ΔNV ^R 13
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	111 19. 112 33. 113 38. 114 35. 115 23. 116 14. 117 15. 118 17. 119 16. 120 8. 121 9. 122 9. 123 3. 124 9. 125 4. 126 6.	2 27.2 8 26.9 3 26.0 4 23.8 9 21.3 3 19.5 7 17.8 5 15.4 6 12.7 5 10.9 1 10.2 1 10.3 3 10.2 7 9.9	74 75 76 77	-0.5 -0.3 -0.9 -2.2 -2.5 -1.8 -1.7 -2.4 -2.7 -1.8 -0.7 +0.1 -0.1 -0.3 -0.3 +0.5	.90 .91 .92 .93 .94 .95 .96 .97 .98	.50 .51 .52 .53 .54 .55 .55 .56 .57 .58 .59 .60 .61	
OCT 1964	0 6. 1 7. 2 15. 3 17. 4 14. 5 11. 6 6. 7 24. 8 15. 9 11. 10 16. 12 20. 13 15. 14 17. 15 28. 16 25. 18 48. 19 45. 20 47. 21 56. 22 51. 23 50. 24 57. 25 70. 27 110. 28 93. 29 111. 30 69. 31 86. 32 97. 33 91.	4 10.1 11.0 5 11.7 2 12.0 7 12.5 8 13.6 1 14.6 9 15.5 9 16.4 17.4 1 19.7 8 22.3 0 24.5 2 27.7 4 31.3 3 34.5 7 37.4 3 40.7 7 44.7 7 50.3 2 56.7 2 70.2 4 72.7 9 75.0 78.8 8 82.2 84.6 5 87.5 3 91.3	-48 -47 -46 -45 -44 -43 -42 -41 -40 -39 -38 -35 -34 -33 -32 -31 -30 -29 -28	+0.5 +1.1 +1.0 +0.4 +0.5 +0.9	.01 .02 .03 .04 .04 .05 .06 .07 .08 .09 .09 .10 .11 .11	.62 .63 .63 .64 .65 .66 .67 .68 .69 .70 .71 .72 .73 .73 .74 .75 .77 .77 .78 .79 .80 .81 .82 .83 .84 .84 .85 .86 .87 .88	R _{MIN} SCN 20

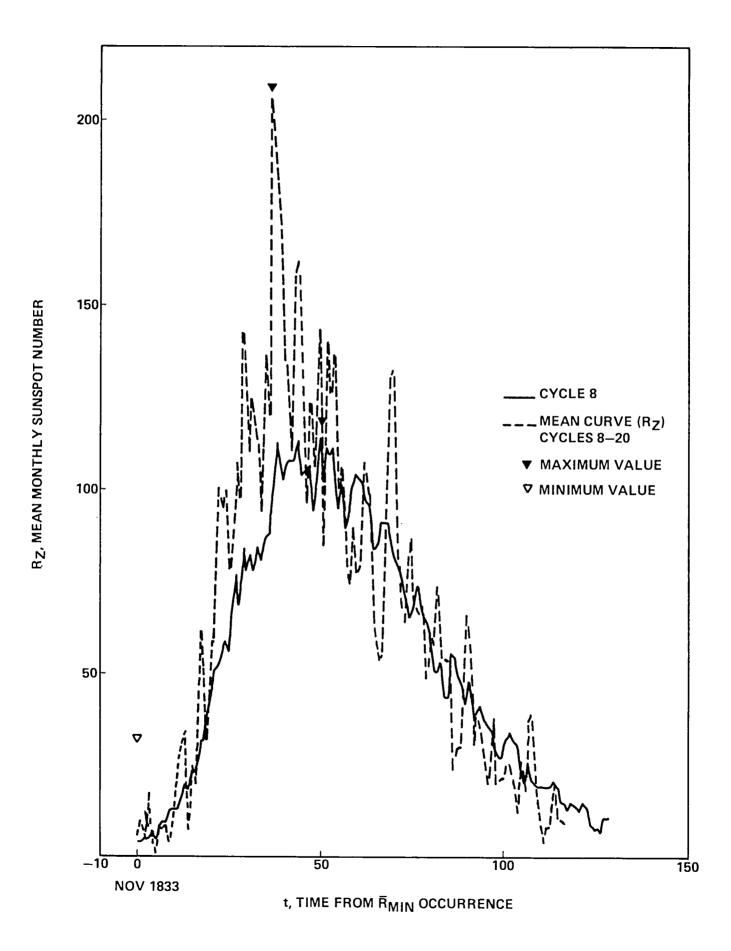
NOV 1968	34 107.2 35 76.8 36 88.2 37 94.3 38 126.4 39 121.8 40 111.9 41 92.2 42 81.2 43 127.2 44 110.3 45 96.1 46 109.3 47 117.2 48 107.7 49 86.0 50 109.8 51 104.4 52 120.5 53 135.8 54 106.8 55 120.0 56 106.0 57 96.8 58 98.0 59 91.3 60 95.7 61 93.5	95.3 95.3 95.0 97.1 100.6 102.6 102.6 107.2 107.6 106.6 105.2 104.8 107.0 109.9 110.6 110.1 110.0 109.6 108.0 106.4 106.2 106.5 105.9 106.5 105.4 104.1 104.6	-15 -14 -13 -12 -11 -10 - 8 - 6 - 5 - 4 - 2 - 1 - 1 - 2 - 3 - 4 - 1 - 1 - 2 - 3 - 4 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1	0.0 -0.3 +2.1 +3.5 +2.0 +0.3 +1.8 +2.5 +0.4 -1.0 -1.4 -0.4 +2.2 +2.9 +0.7 -0.5 -0.1 -0.6 -1.6 -1.6 -1.6 -1.6 -1.3 +0.3 +0.3	.24 .25 .26 .27 .28 .29 .30 .31 .32 .33 .34 .35 .36 .37 .38 .39 .41 .41 .42 .43 .44	.88 .89 .90 .91 .91 .92 .93 .94 .95 .95 .96 .97 .98 .99 .00 .01 .02 .03 .04 .05 .06 .07 .08 .09	R _{MAX}
	62 97.9 63 111.5 64 127.8 65 102.9 66 109.5 67 127.5 68 106.8 69 112.5 70 93.0 71 99.5 72 86.6 73 95.2 74 83.5 75 91.3 76 79.0 77 60.7 78 71.8 79 57.5 80 49.8 81 81.0 82 61.4 83 50.2 84 51.7	104.9 105.6 106.0 106.2 106.1 105.8 105.3 103.8 101.0 97.2 93.9 89.4 84.1 80.4 77.8 74.4 70.9 68.1 66.7 65.4 64.6 65.8 66.2	13 14 15 16 17 18 19 20 21 22 23 24 25 27 28 29 31 32 33 34 35	+0.7 +0.4 +0.2 -0.1 -0.3 -0.5 -1.5 -2.8 -3.8 -3.3 -4.5 -5.3 -3.7 -2.6 -3.4 -3.5 -2.8 -1.4 -1.3 -0.8 +1.2 +0.4 +0.6	.44 .45 .46 .47 .49 .49 .51 .51 .52 .53 .54 .55 .56 .57 .59 .50	.10 .11 .12 .13 .14 .15 .16 .17 .17 .18 .19 .20 .20 .21 .22 .23 .24 .25 .26 .26	△ÑV ^R 13

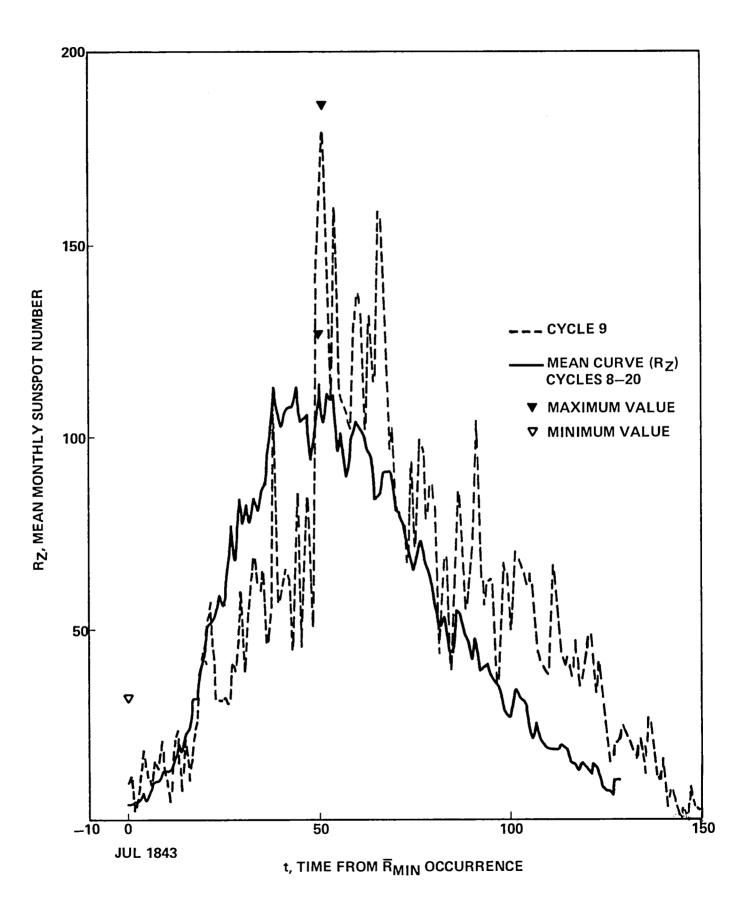
	136 4.3 137 21.9 138 18.8 139 12.4 140 12.2	13.2 12.2 12.6 12.5 12.2	87 88 89 90 91	-1.0 +0.4 -0.1 -0.3 +0.7	.97 .98 .99 .99	.65 .66 .67 .68	
JUN 1976	0 12.2 1 1.9 2 16.4 3 13.5 4 20.6 5 5.2 6 15.3 7 16.4 8 23.1 9 8.7 10 12.9 11 18.6 12 38.5 13 21.4 14 30.1 15 44.0 16 43.8 17 29.1 18 43.2 19 93.6 21 76.5 22 99.7 23 82.7 24 95.1 25 70.4 26 58.1 27 138.2 28 125.1 29 97.9 30 122.7	12.2 12.9 14.0 14.3 13.5 13.5 14.8 16.7 18.1 20.0 22.2 24.2 26.3 29.0 33.4 39.1 45.6 51.9 56.9 61.3 64.5 69.6 76.9 83.2 89.3 97.4 104.0 108.4 111.1 113.3 117.7 123.7	-42 -41 -40 -39 -38 -37 -36 -35 -31 -39 -28 -27 -26 -25 -24 -22 -21 -20 -18 -16 -15 -14 -12	+0.7 +1.1 +0.8 0.0 +1.9 +1.9 +1.4 +1.9 +2.0 +2.1 +2.1 +2.7 +4.4 +5.3 +5.0 +4.4 +5.3 +6.1 +6.3 +6.1 +6.4 +2.7 +2.4 +6.9 +7.2	1.00	.68 .69 .70 .71 .71 .72 .73 .74 .75 .76 .77 .77 .78 .80 .80 .81 .82 .83 .84 .85 .86 .87 .88 .89 .90 .91	\bar{R}_{MIN} SCN 21 $\triangle \bar{R}_{PV}$
DEC 1979	31 166.6 32 137.5 33 138.0 34 101.5 35 134.4 36 149.5 37 159.4 38 142.2 39 188.4 40 186.2 41 183.3 42 176.3 43 159.6 44 155.0	123.7 130.9 136.5 141.1 147.3 153.0 155.0 155.4 155.7 157.8 162.3 164.5 163.9 162.6	-11 -10 - 9 - 8 - 7 - 6 - 5 - 4 - 3 - 2 - 1	+7.2 +5.6 +4.6 +6.2 +5.7 +2.0 +0.4 +0.3 +2.1 +4.5 +2.2 -0.6 -1.3 -1.7		.92 .93 .94 .95 .95 .96 .97 .98 .99	$ar{R}_{ extsf{MAX}}$

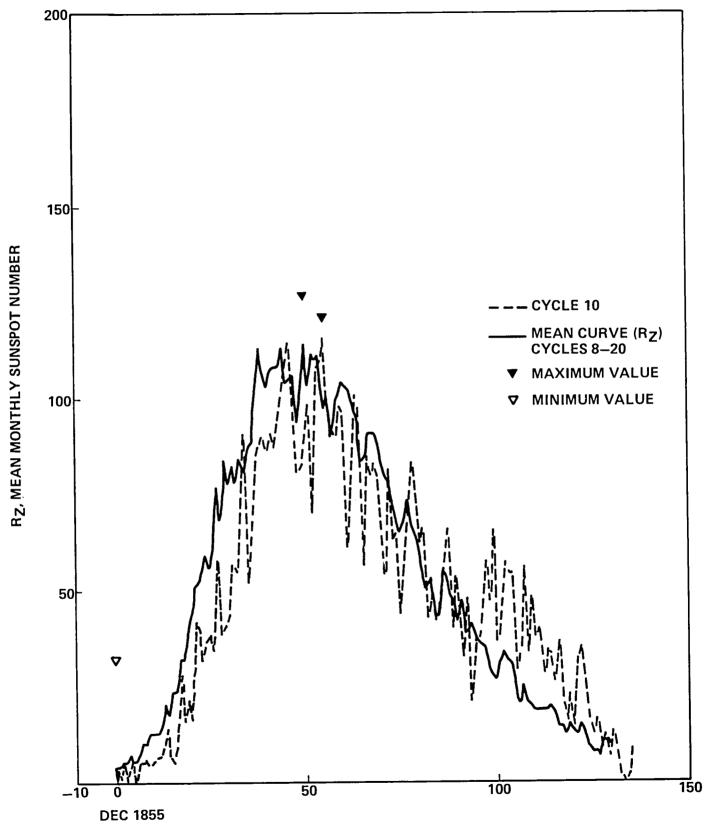
```
-2.2
45 126.2
              160.9
                          3
                                 -2.4
46 164.1
              158.7
                          4
   179.9
              156.3
                          5
                                 -1.6
47
              154.7
                          6
                                 -1.9
   157.3
48
                          7
                                 -2.5
              152.8
49
   136.3
                          8
                                 -0.2
              150.3
50
   135.4
                                 +0.1
                          9
51
   155.0
              150.1
   164.7
              150.2
                         10
                                 -2.5
52
                                 -5.0
53
   147.9
              147.7
                         11
                         12
                                 -2.4
              142.7
   174.4
54
                                 +1.2
                         13
              140.3
55
   114.0
                         14
                                 +1.5
   141.3
              141.5
56
                                 +0.4
   135.5
              143.0
                         15
57
                         16
                                 -0.5
   156.4
              143.4
58
                         17
                                 -1.4
   127.5
              142.9
59
                         18
    90.9
              141.5
60
                         19
   143.8
61
                         20
   158.7
62
                         21
63
   167.3
                         22
   162.4
64
                         23
   137.5
65
66
   150.1
                         24
```

^{*} SCN 14 t(GNV) ACTUALLY OCCURRED AT t = 51, ONLY 2 MONTHS AFTER R(MAX). THE t(GNV) SELECTED IS THE LARGEST GNV CHANGE IN SMOOTHED SUNSPOT NUMBER AFTER t = 51, BEING MORE IN LINE WITH THE OTHER CYCLES; THAT IS, OCCURRING ABOUT 6 OR 7 YEARS INTO THE CYCLE.

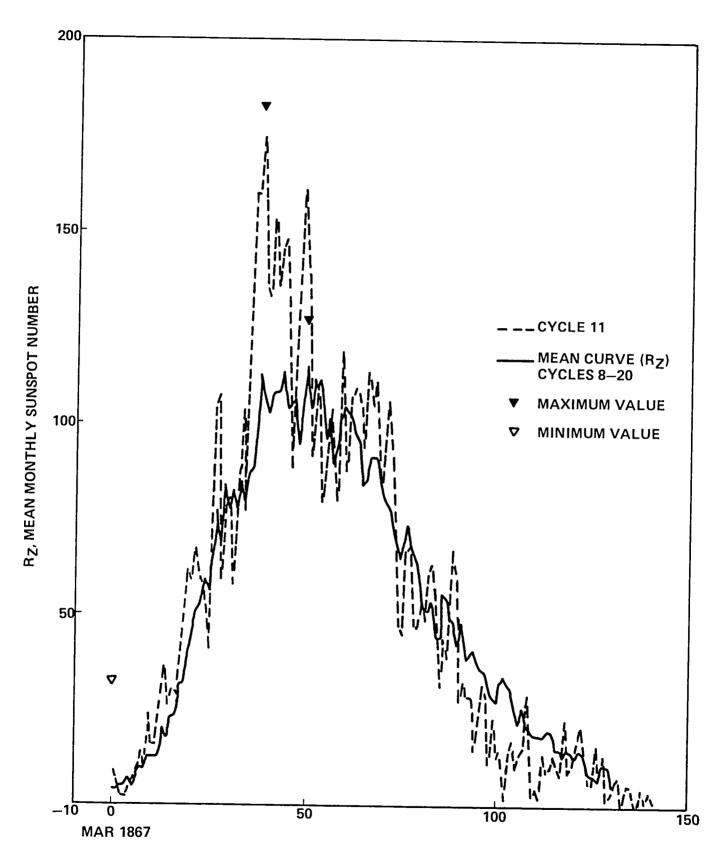
APPENDIX B



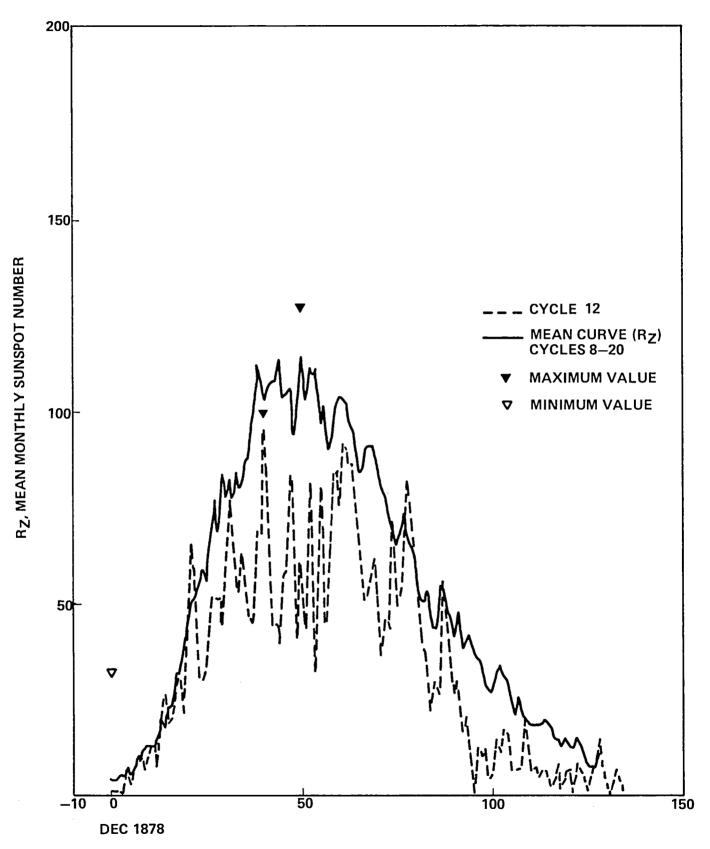




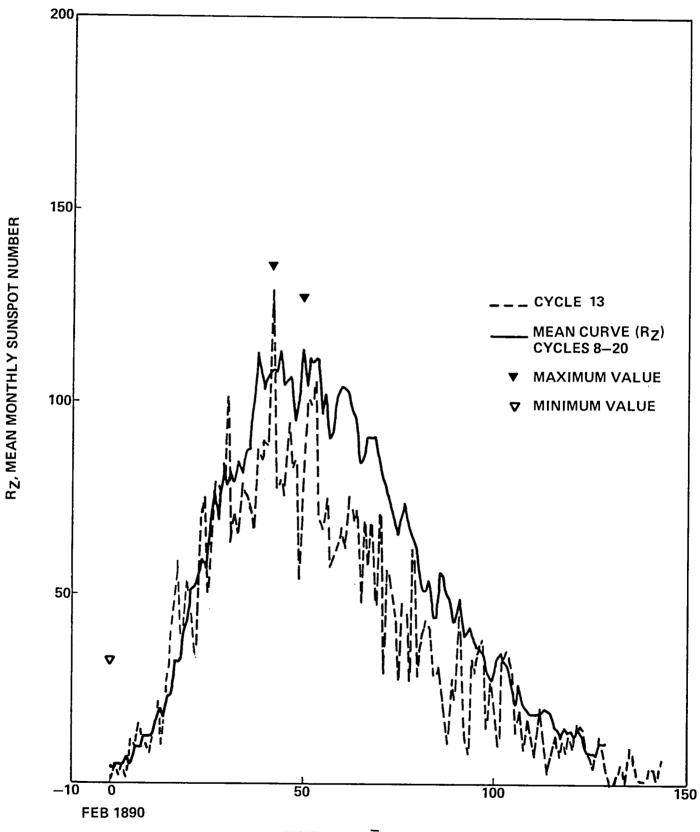
t, TIME FROM $\overline{R}_{\mbox{\footnotesize{MIN}}}$ OCCURRENCE



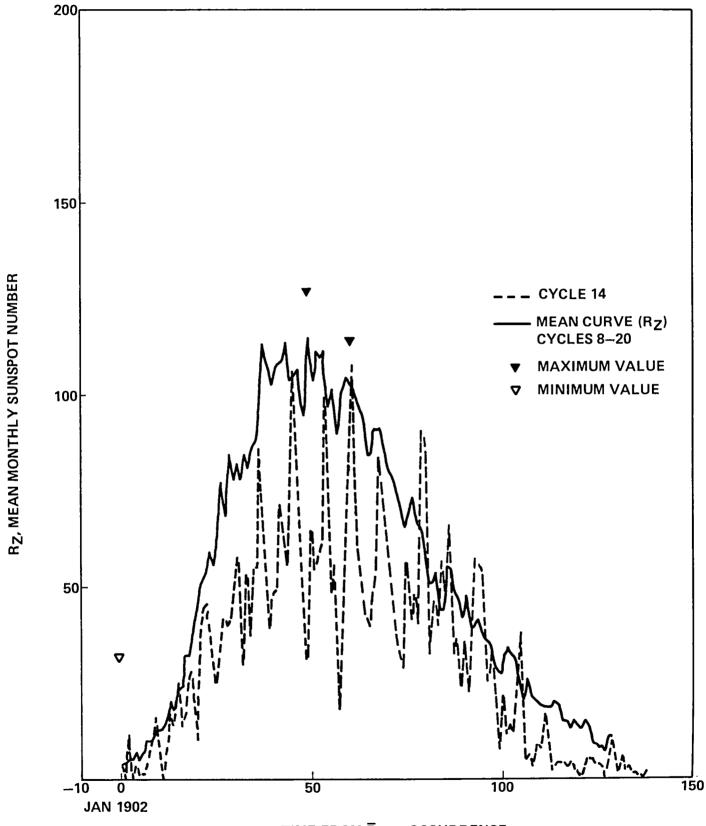
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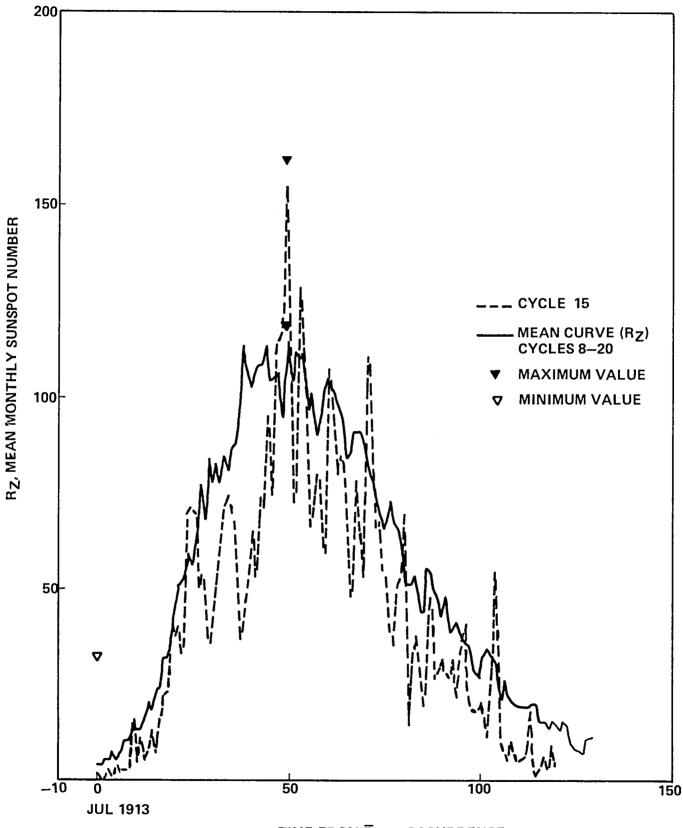
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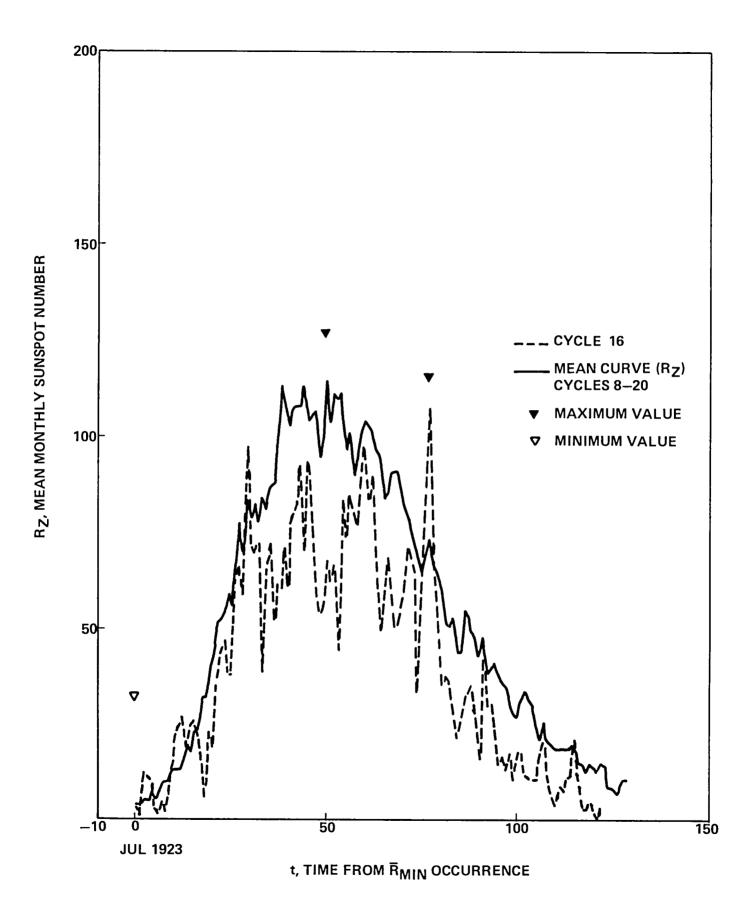
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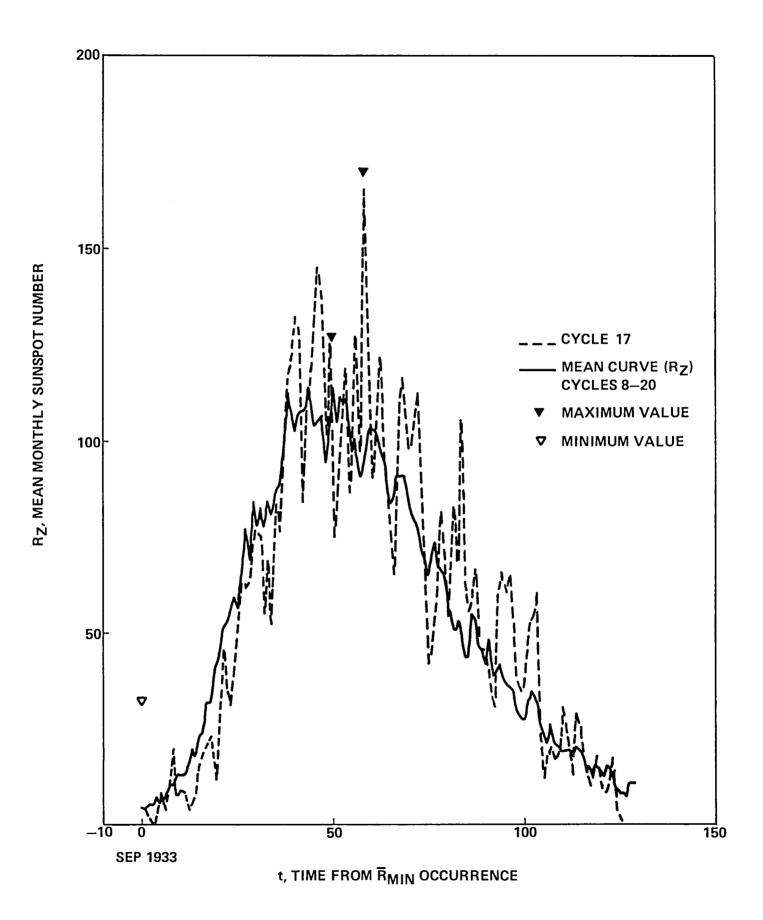


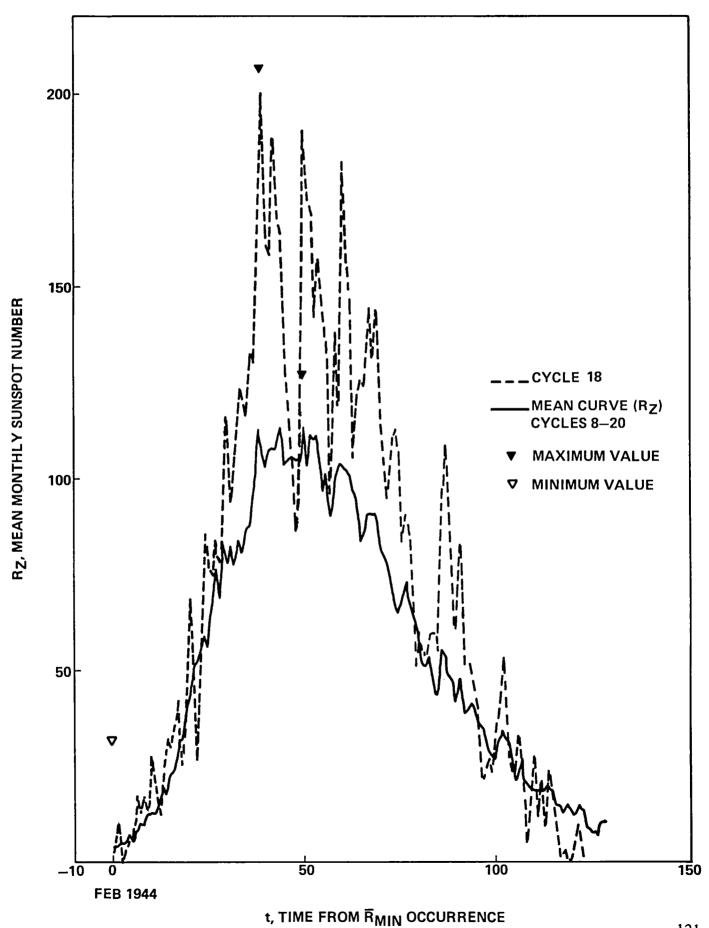
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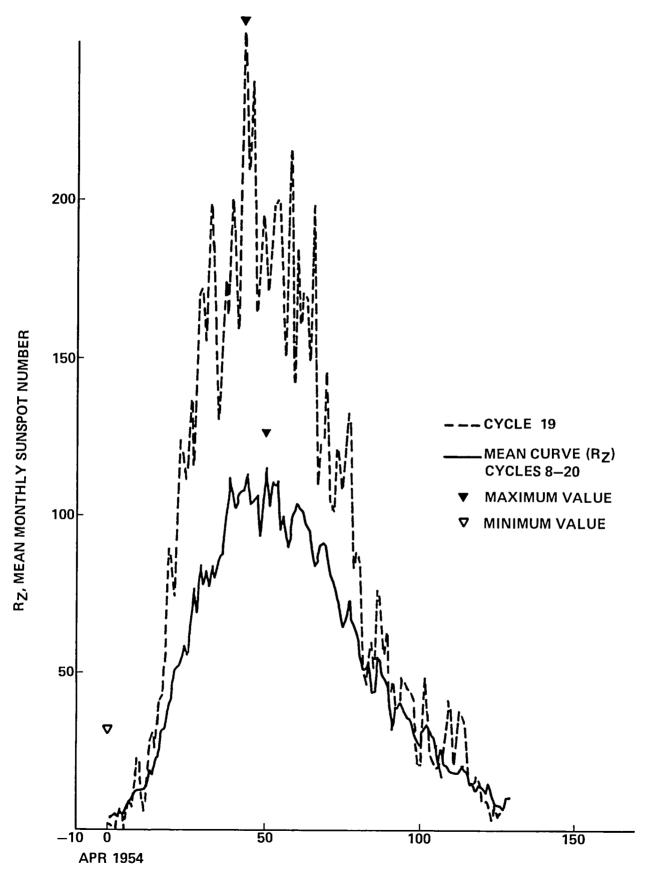


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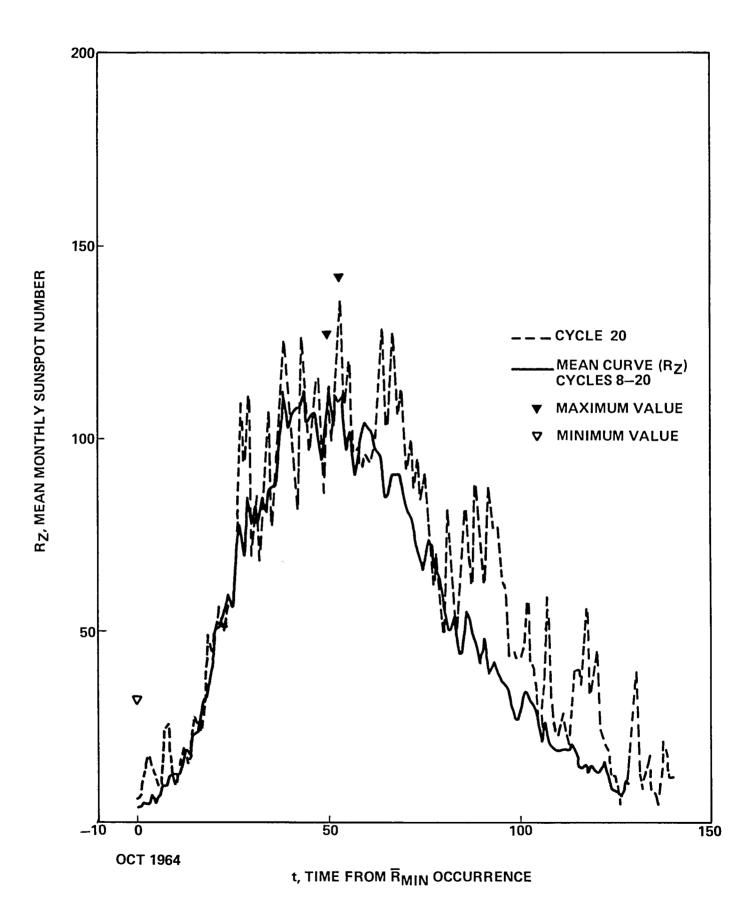




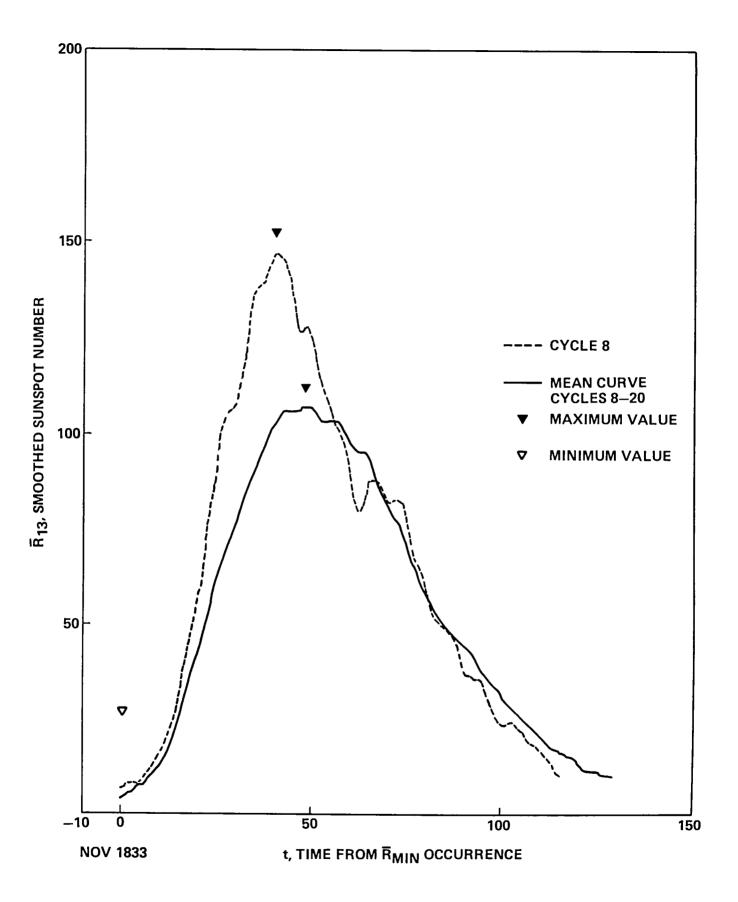


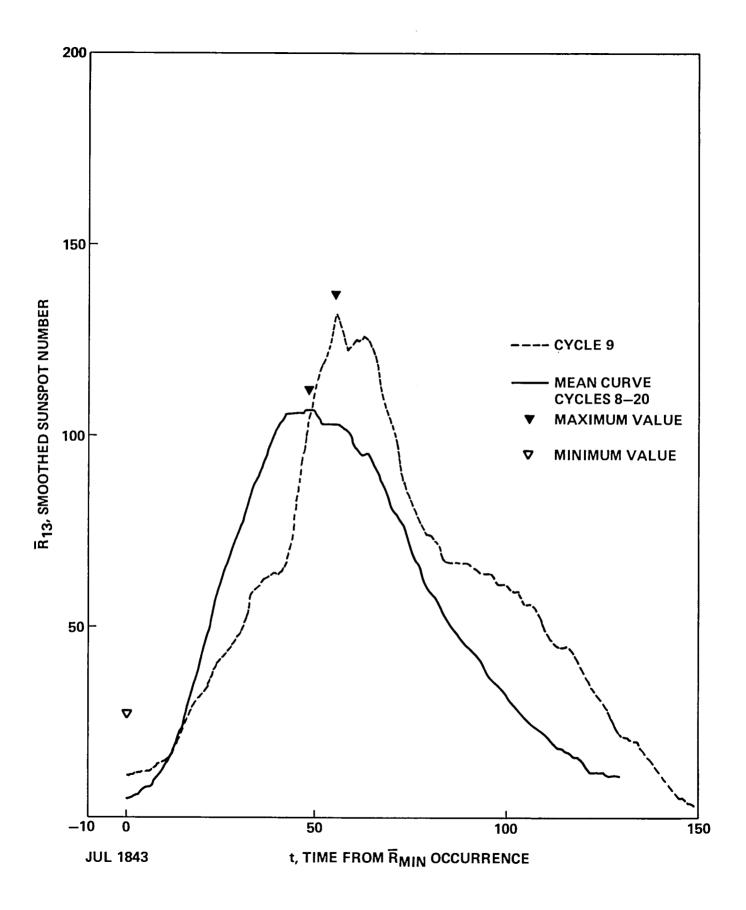


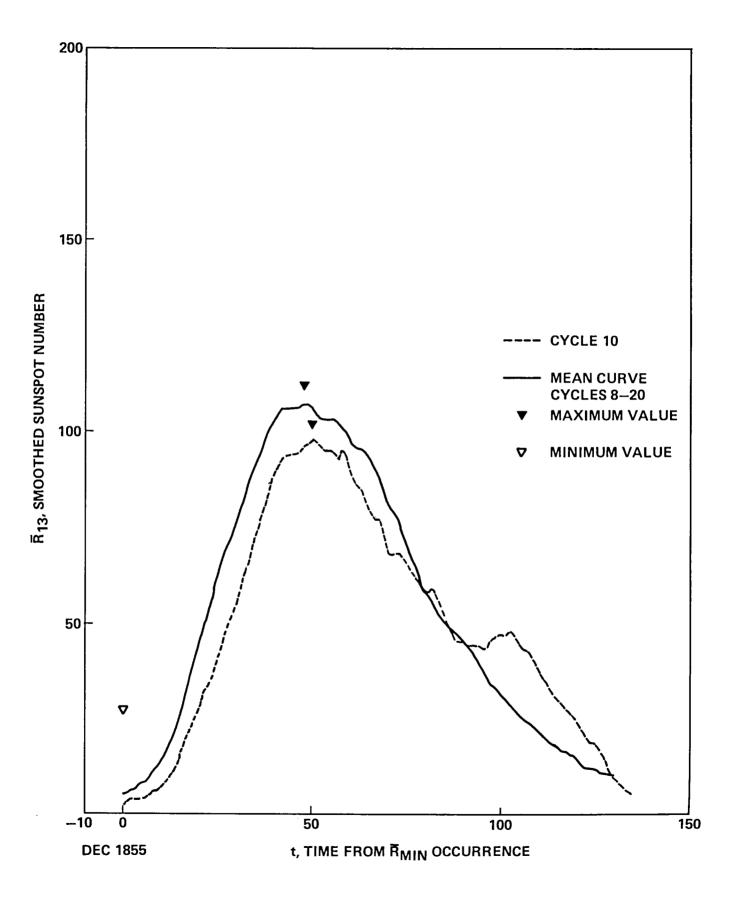
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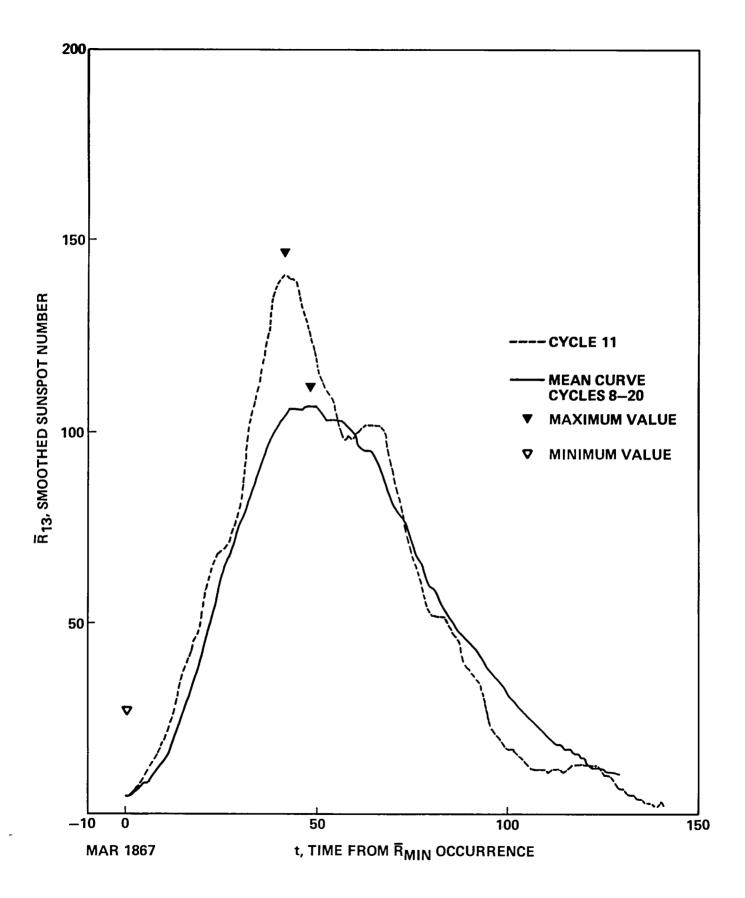


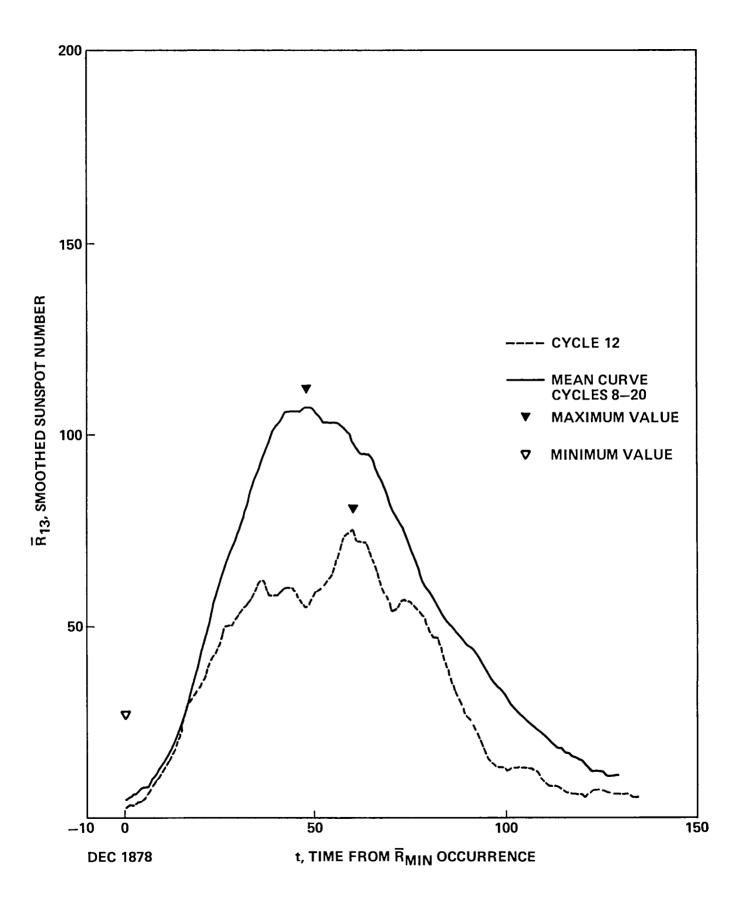
APPENDIX C

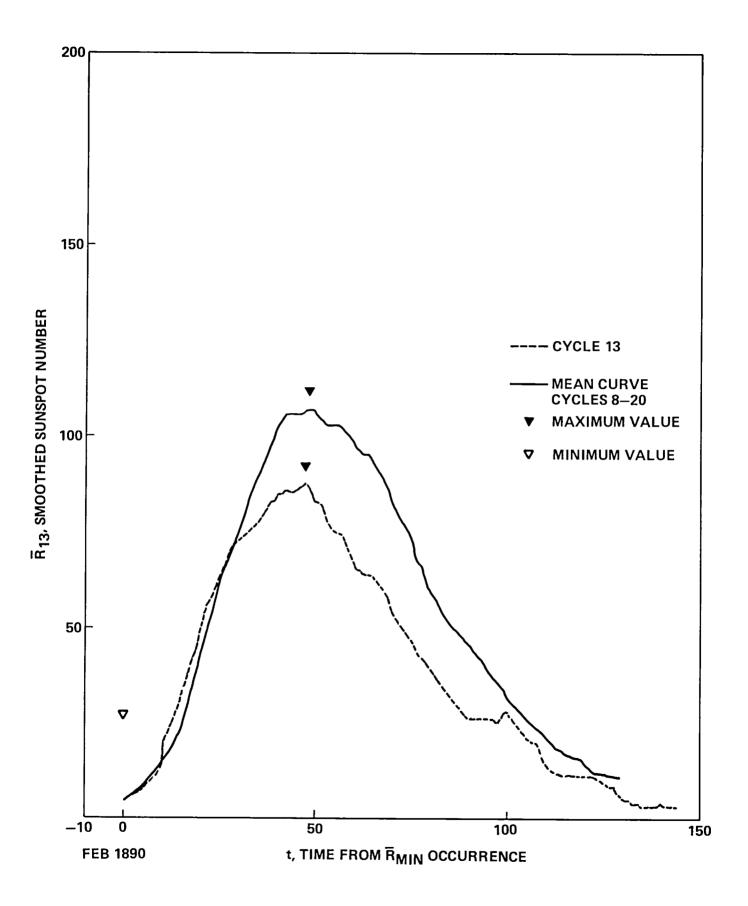


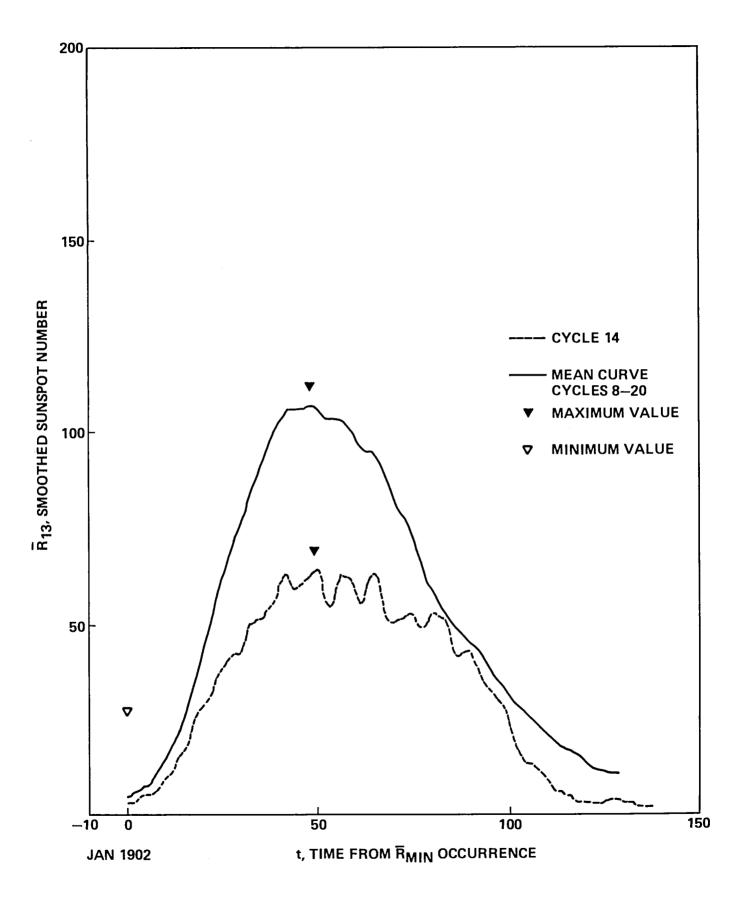


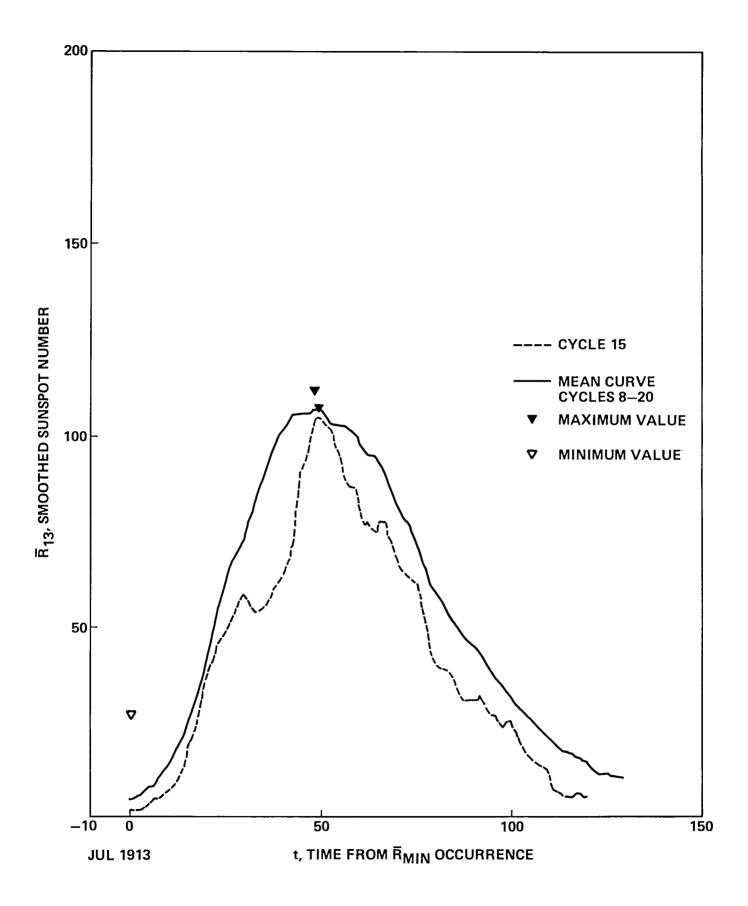


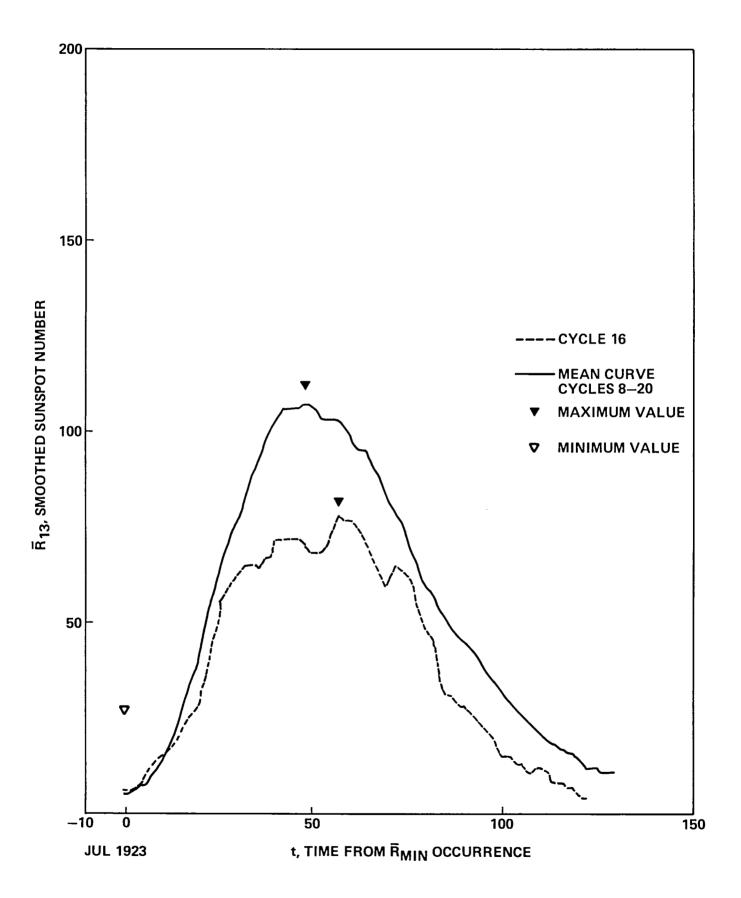


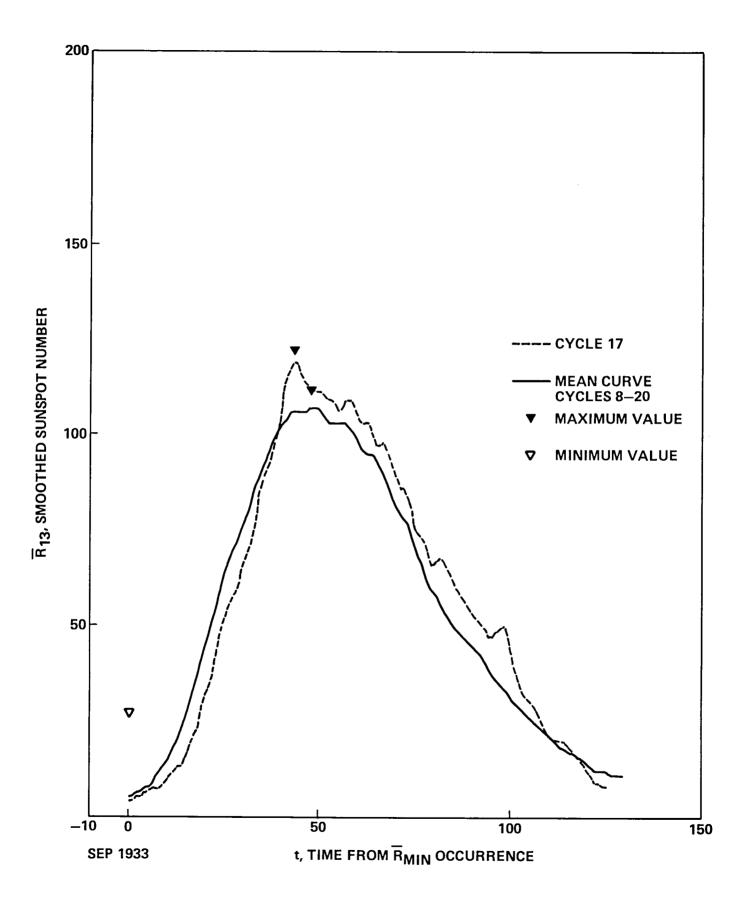


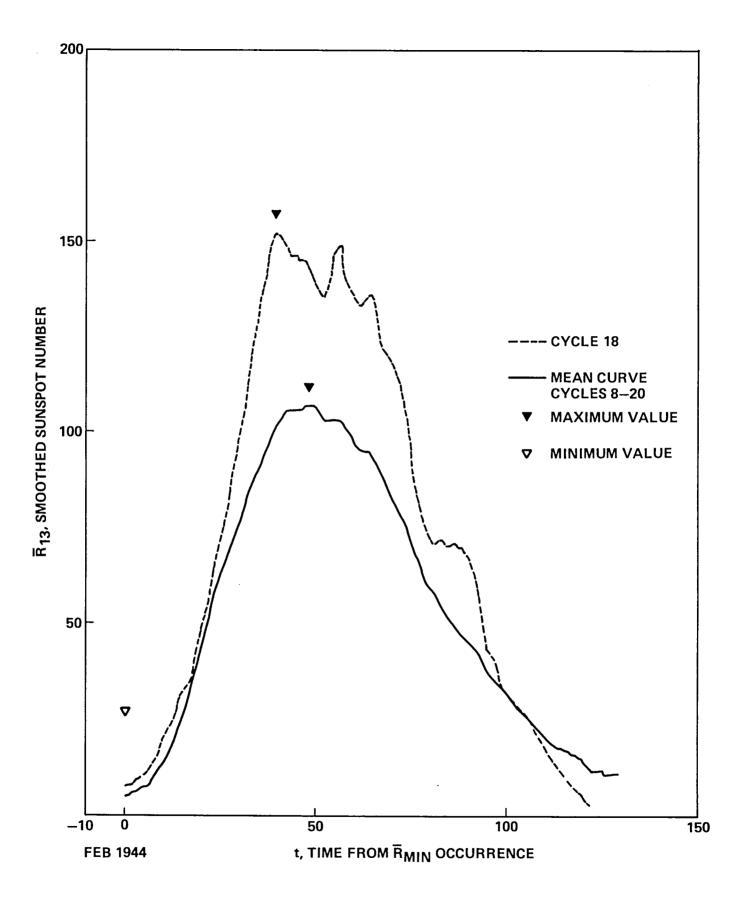


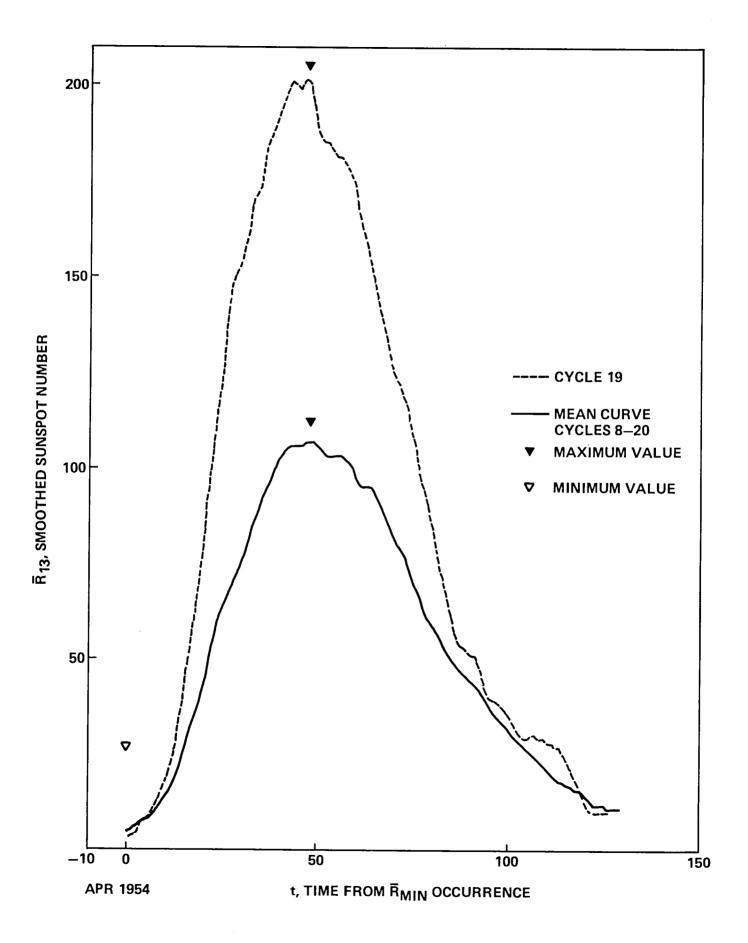


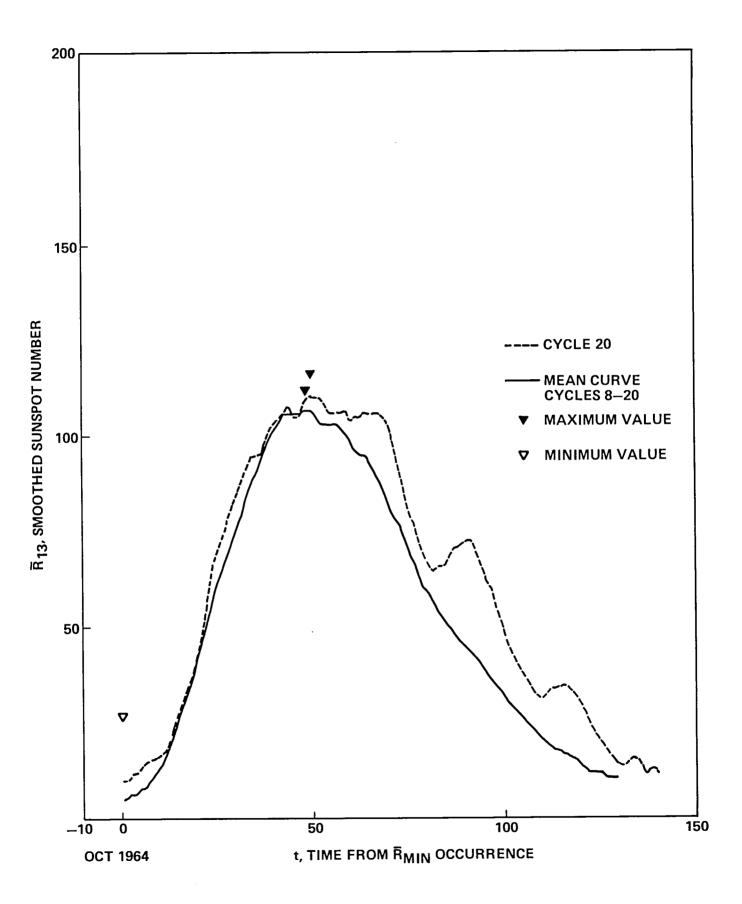




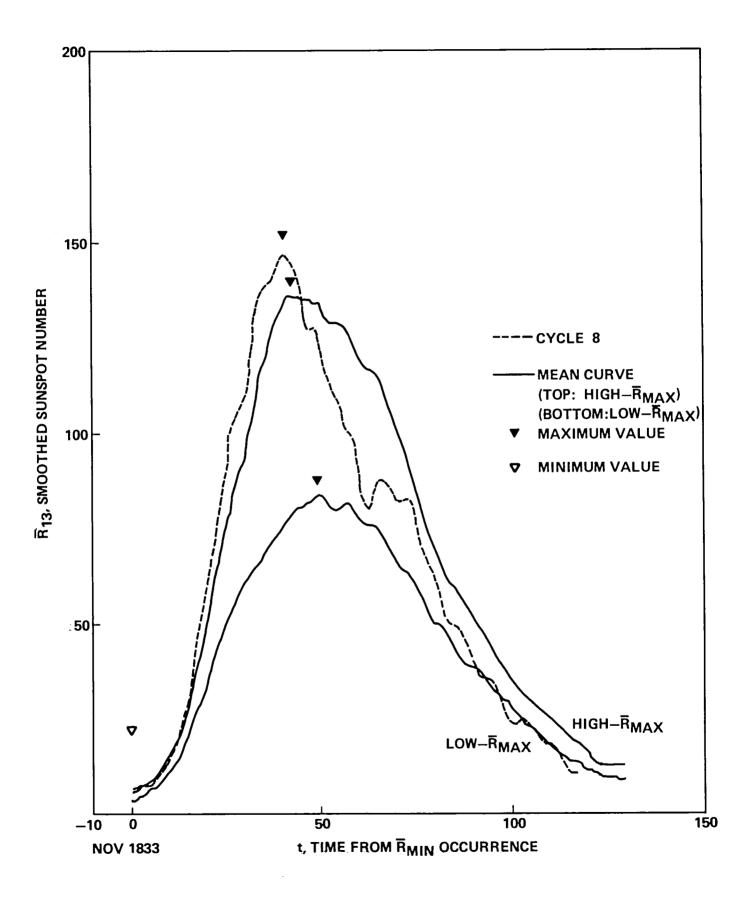


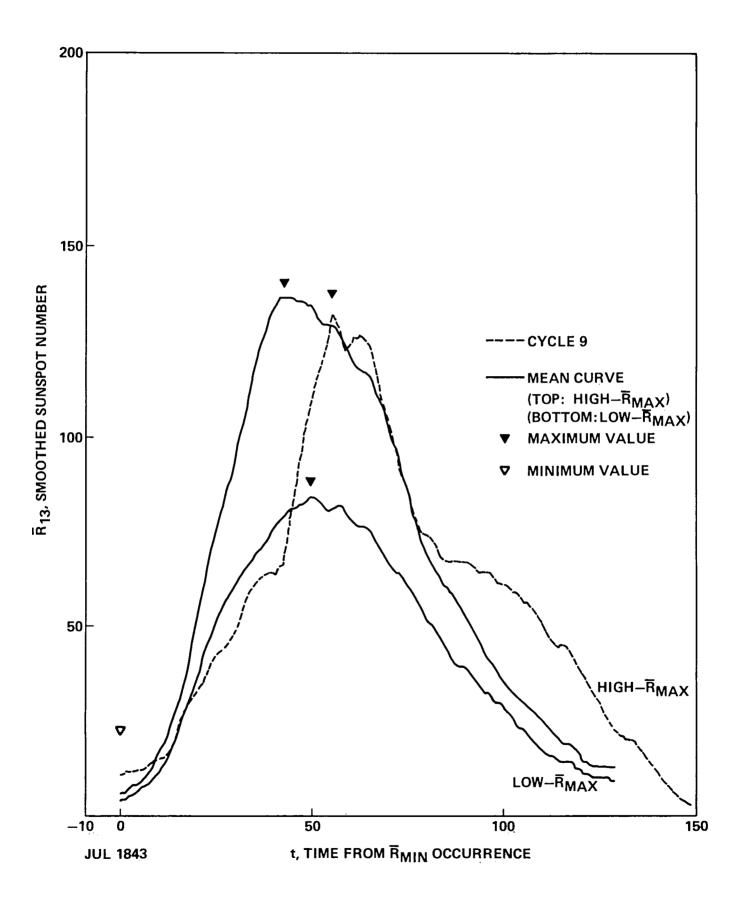


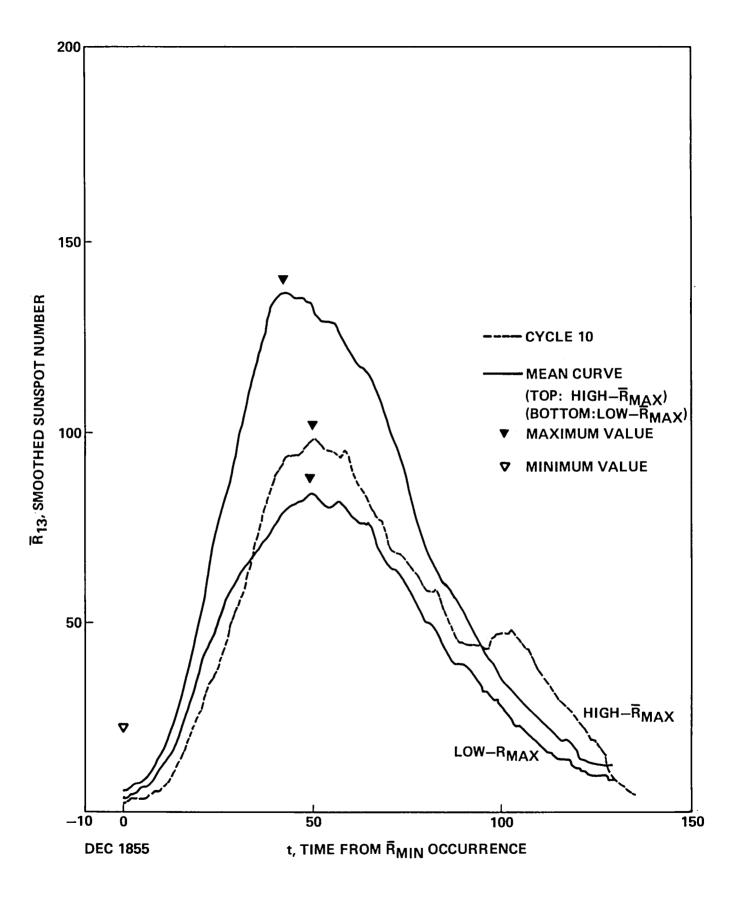


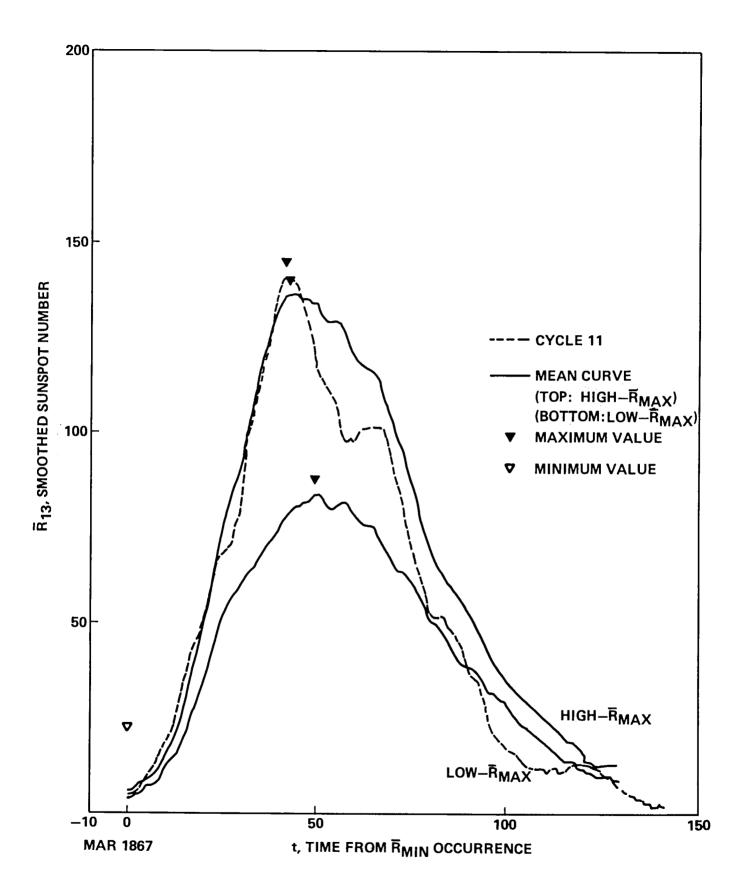


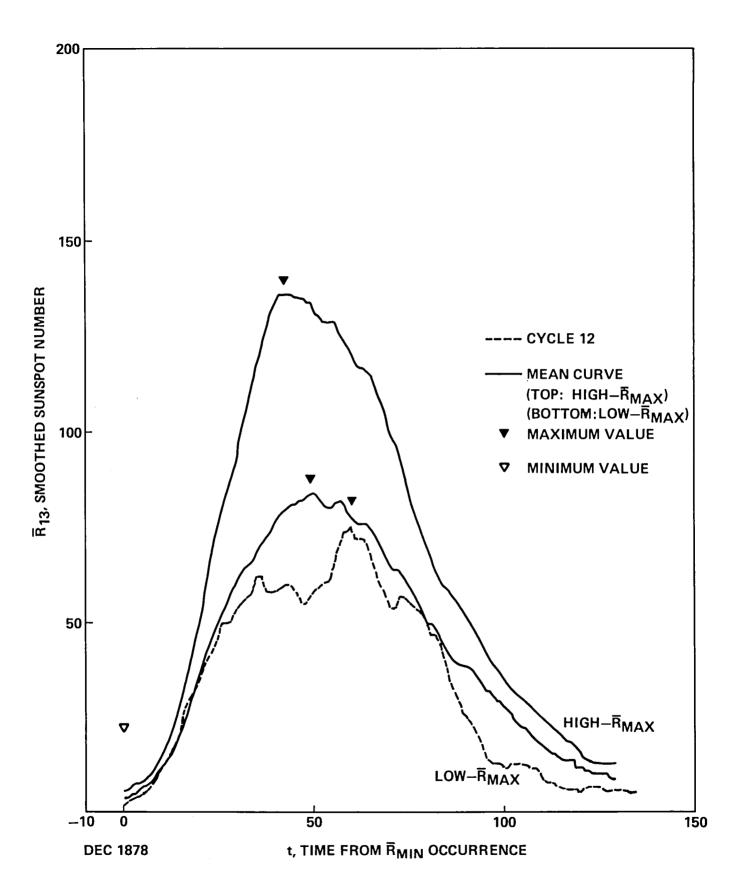
APPENDIX D

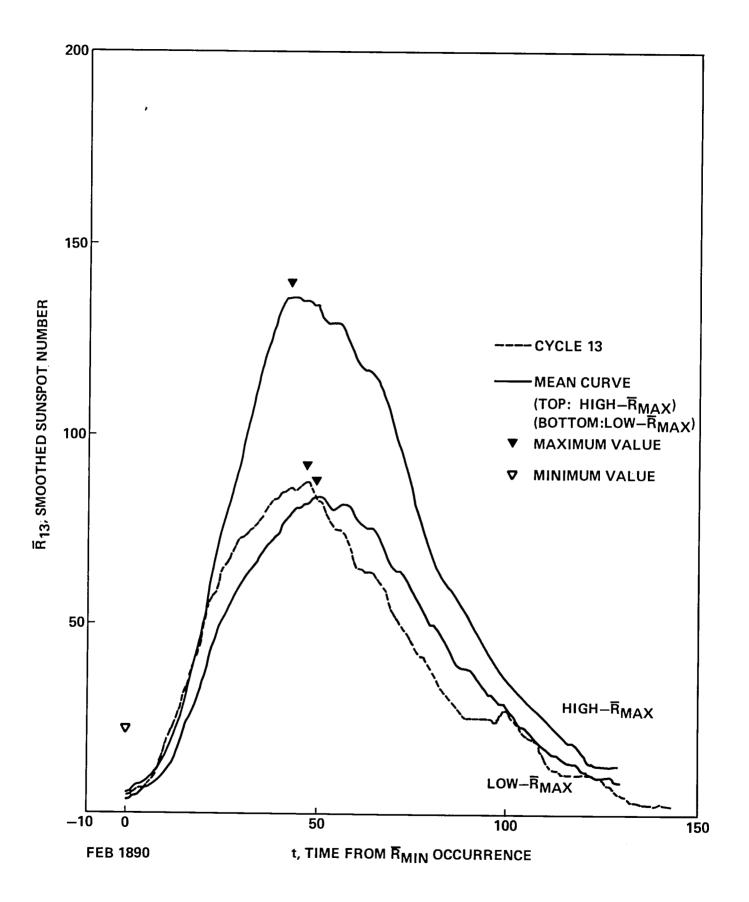


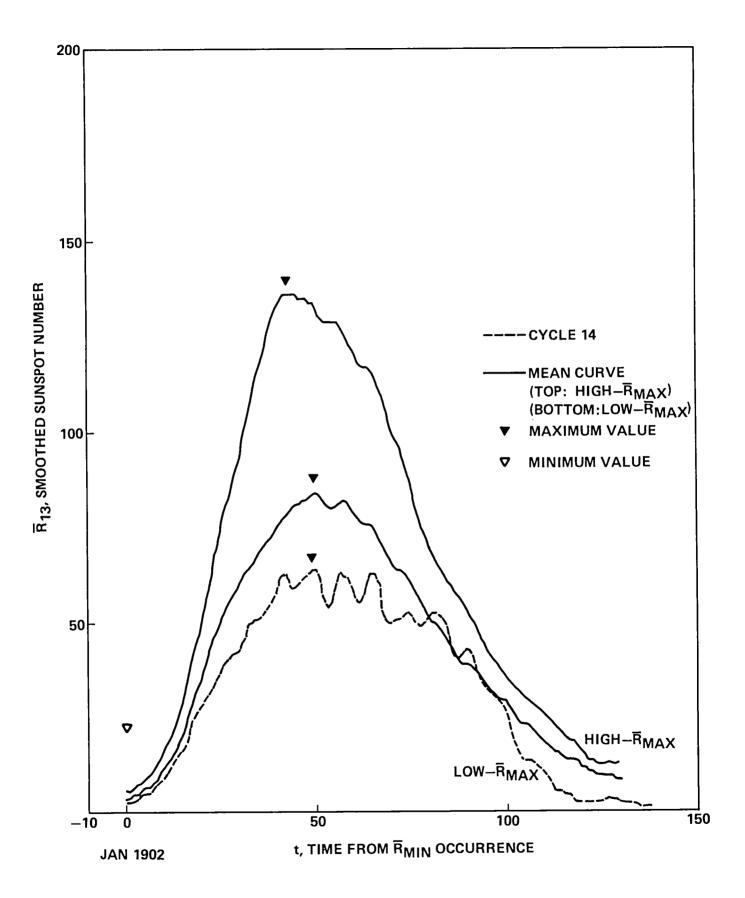


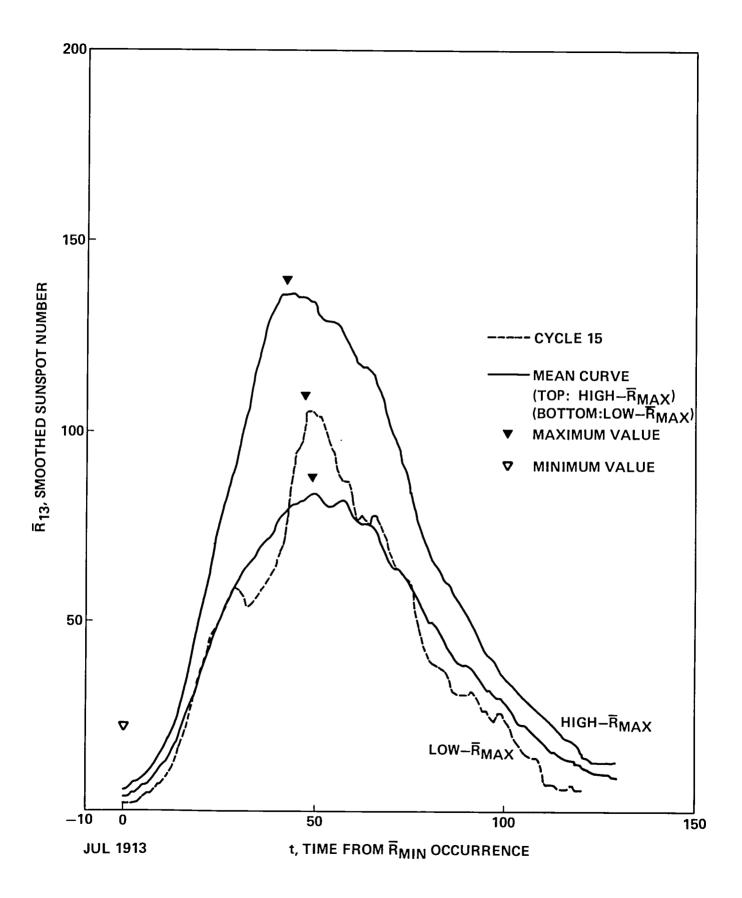


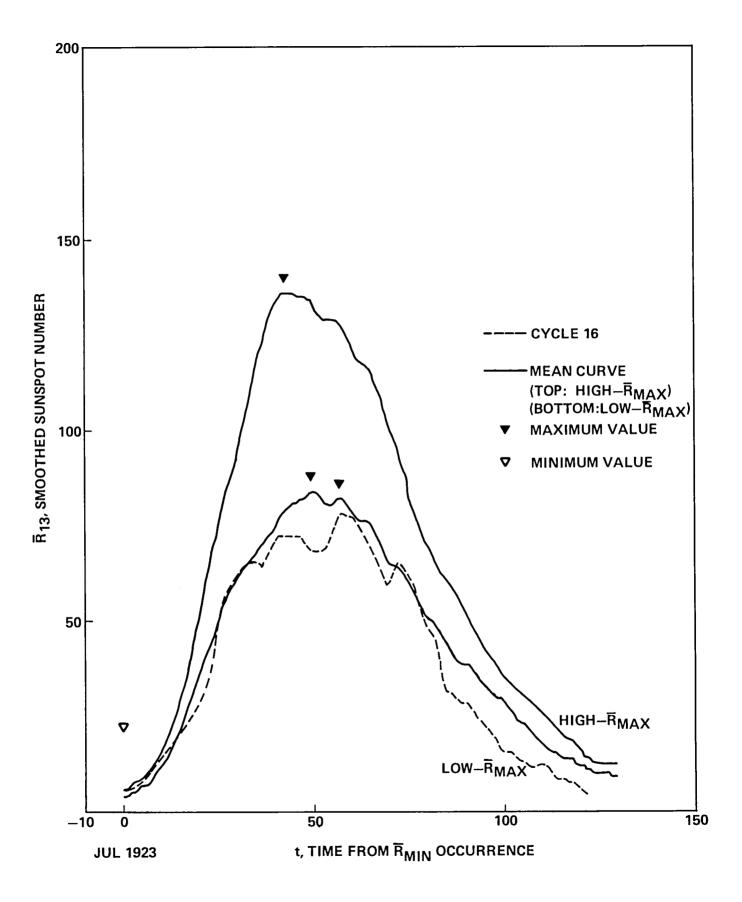


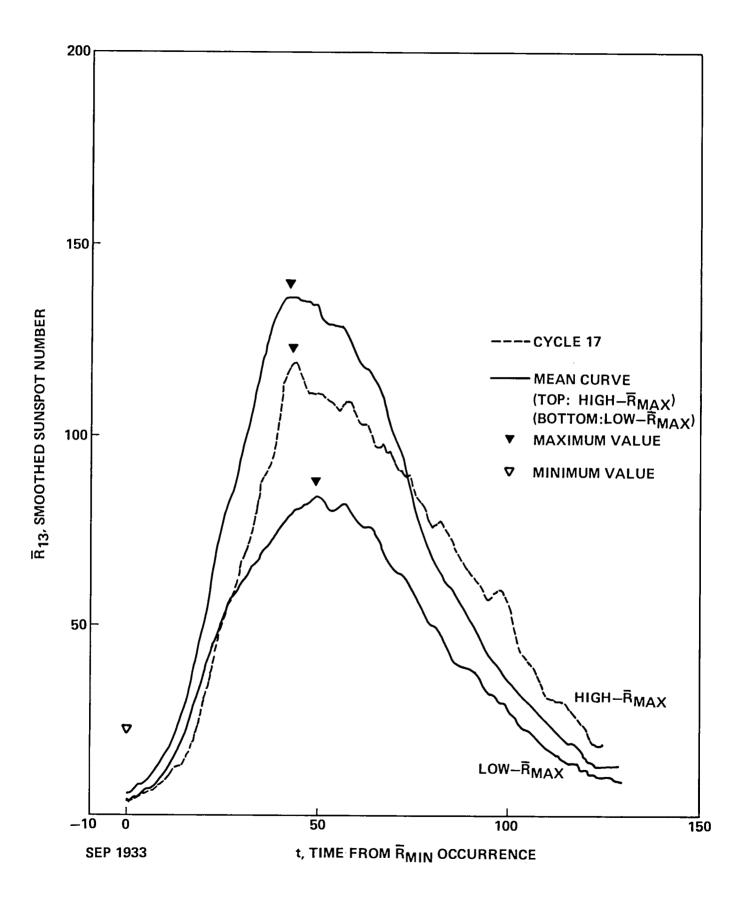


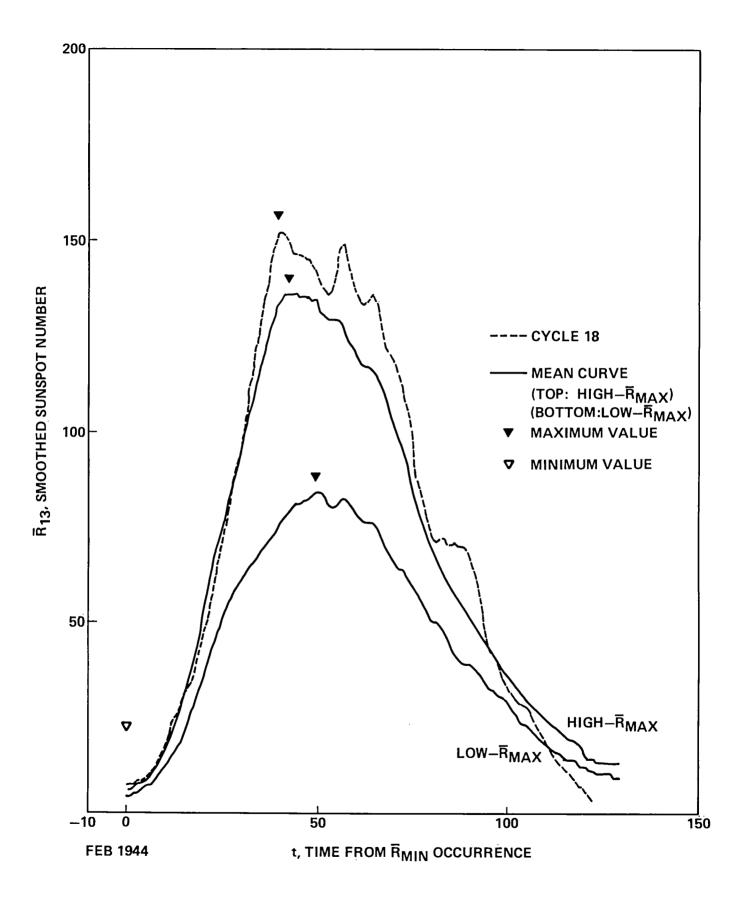


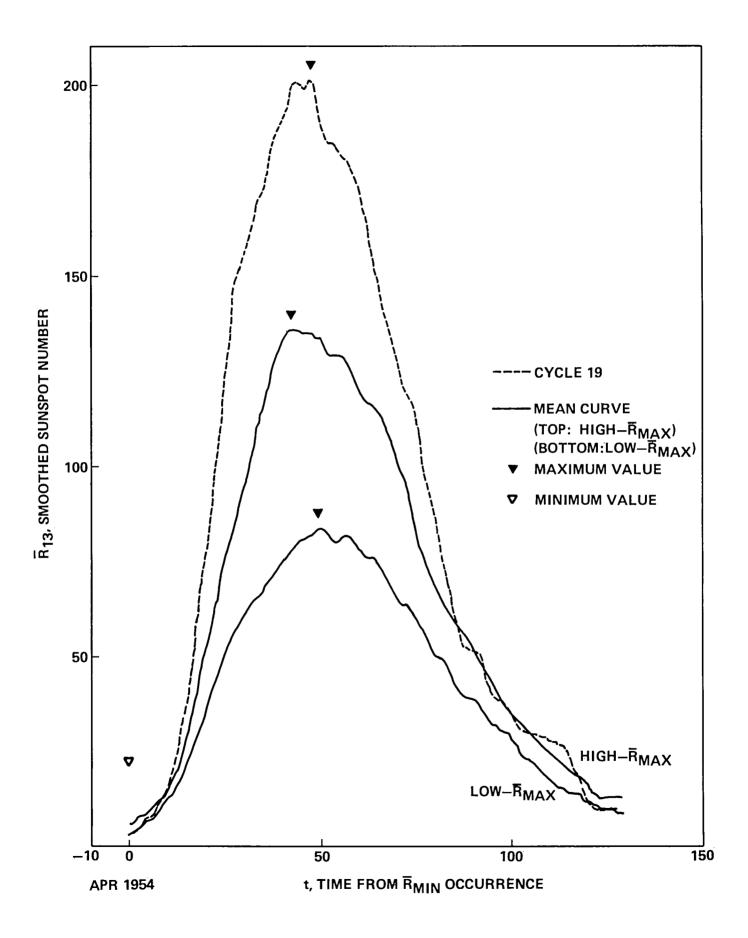


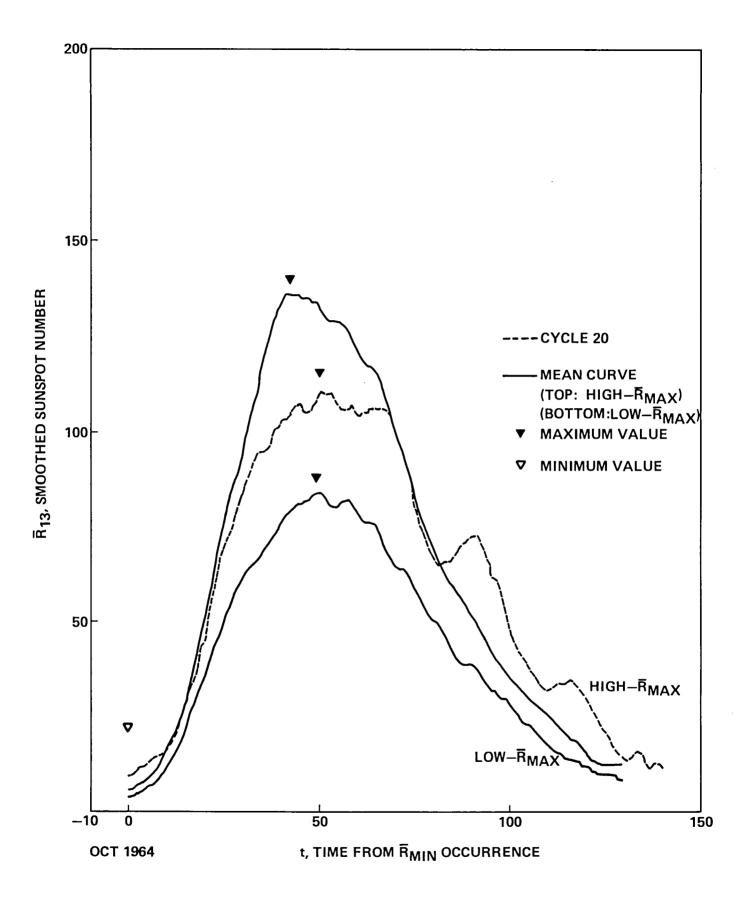




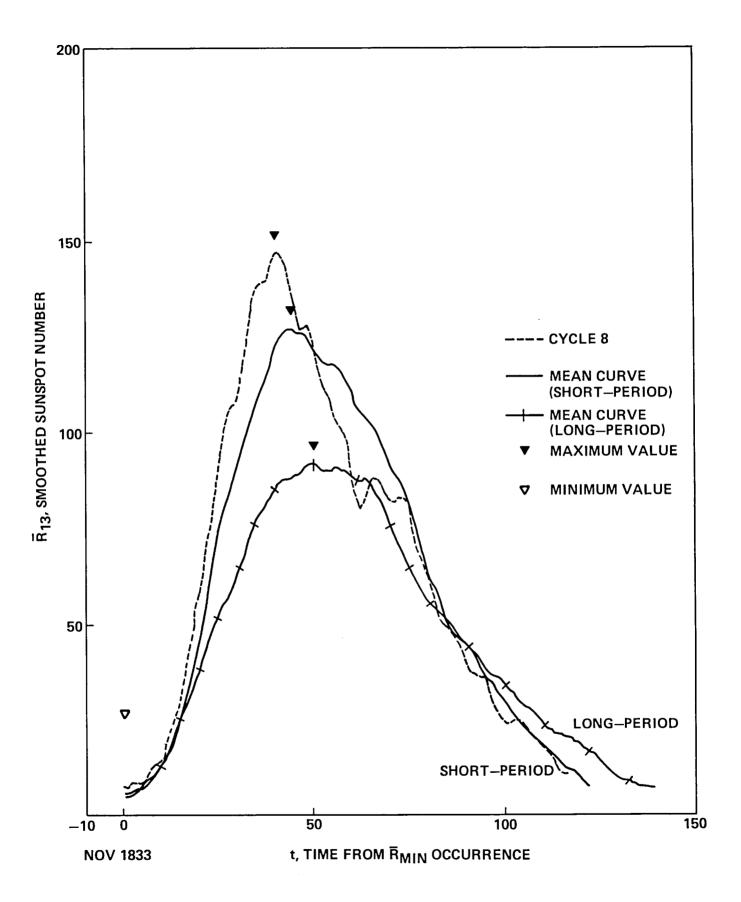


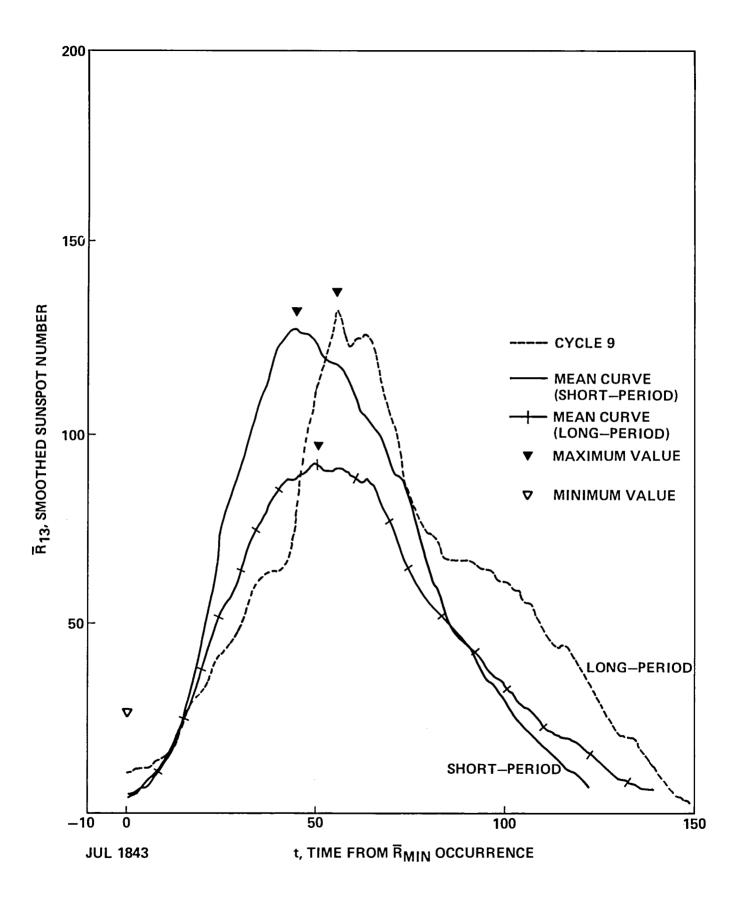


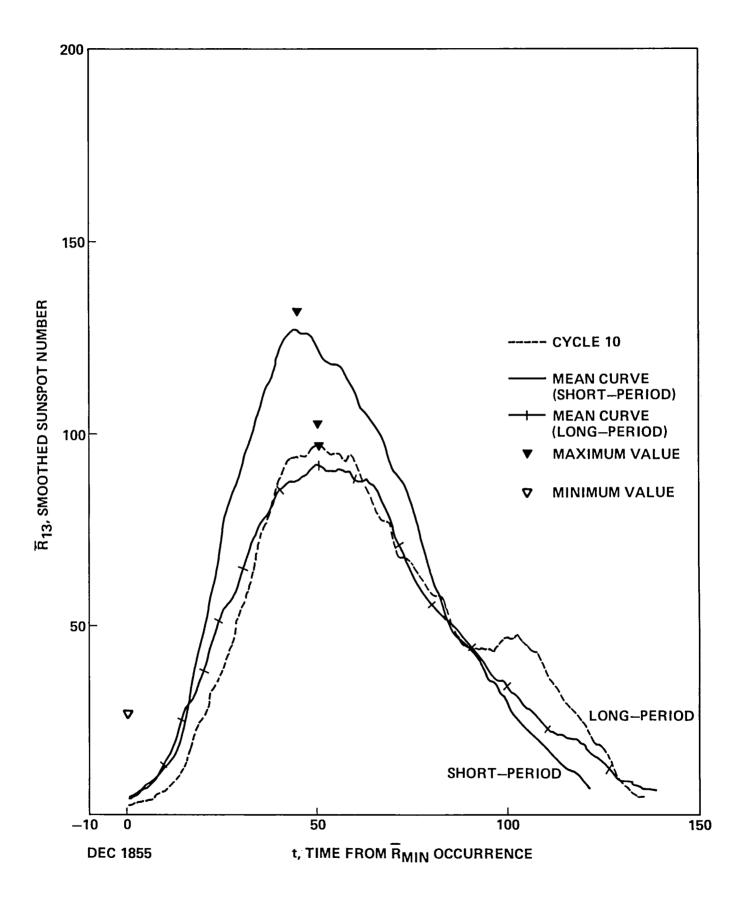


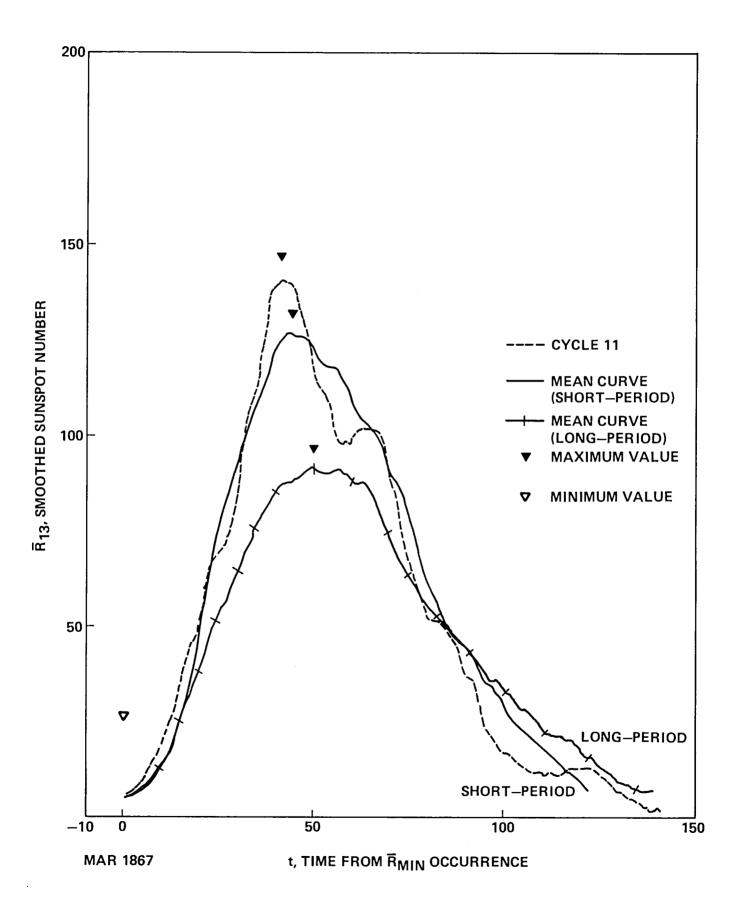


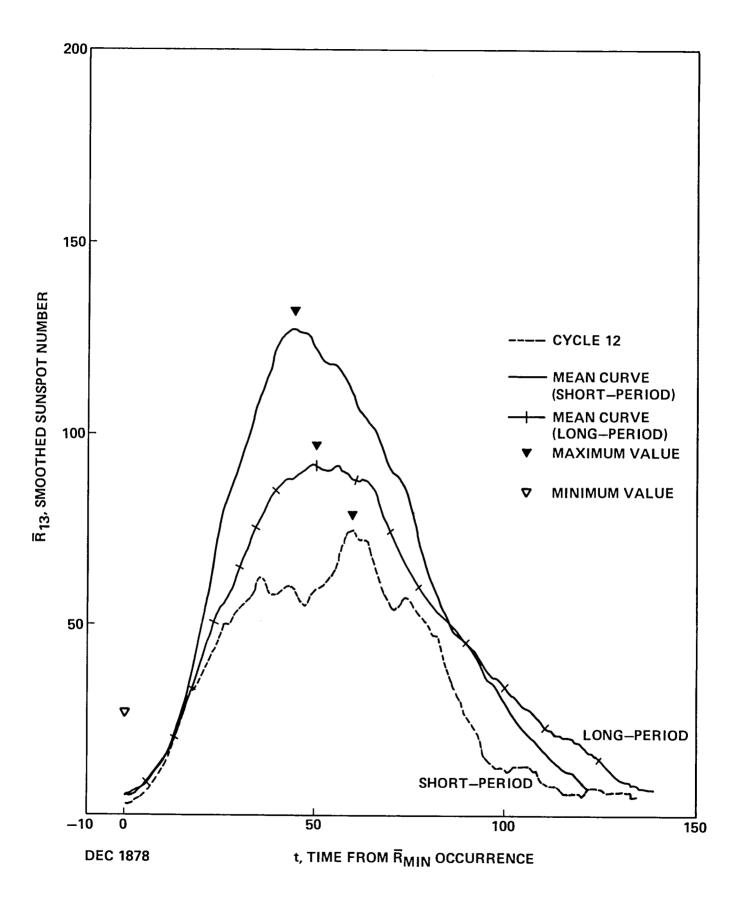
APPENDIX E

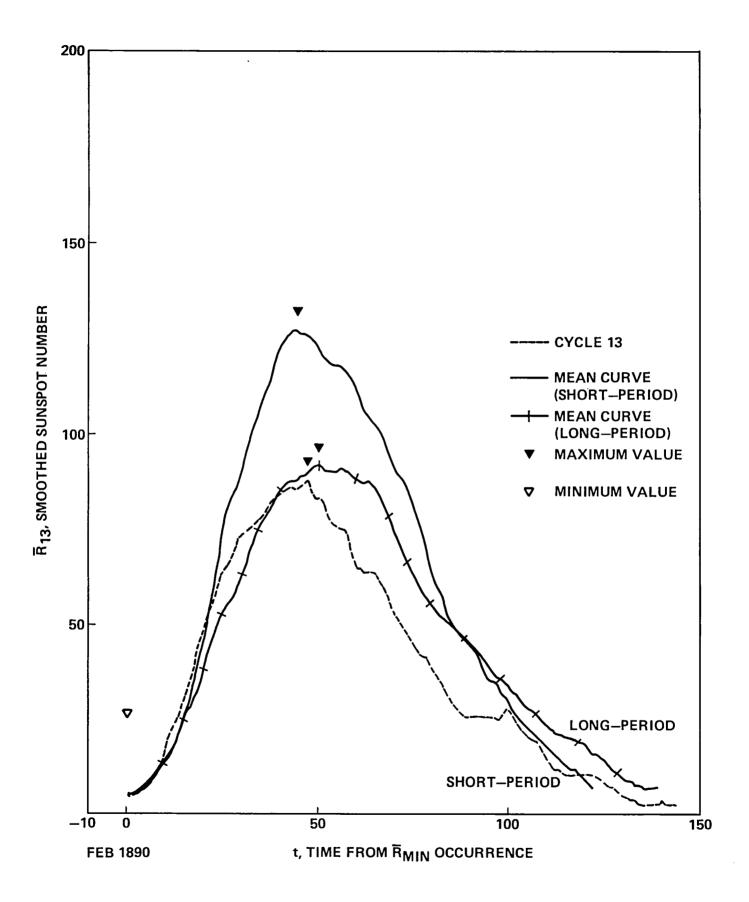


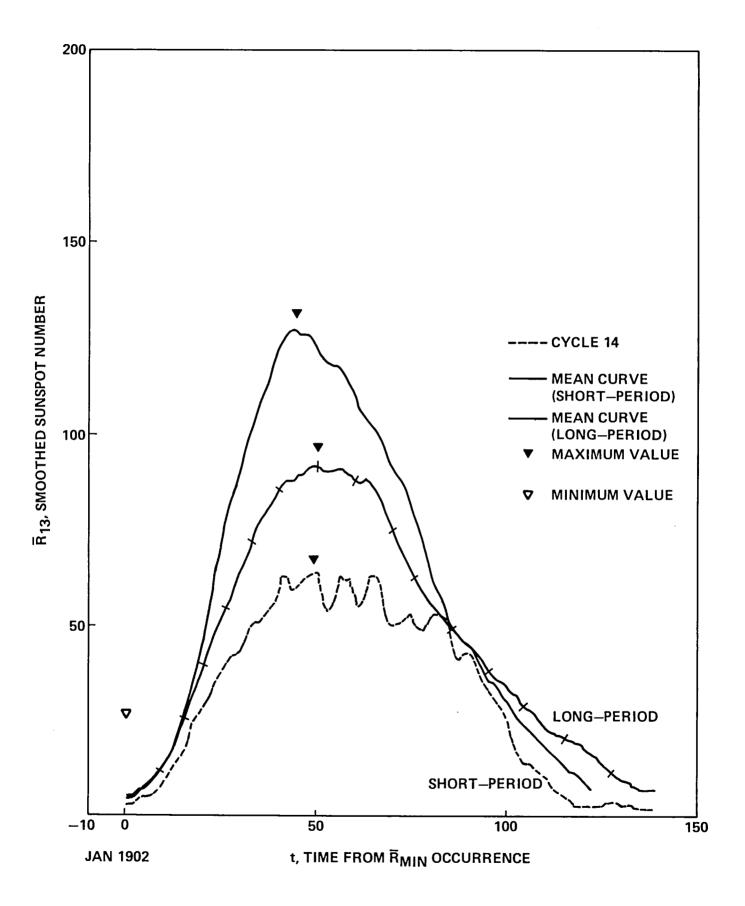


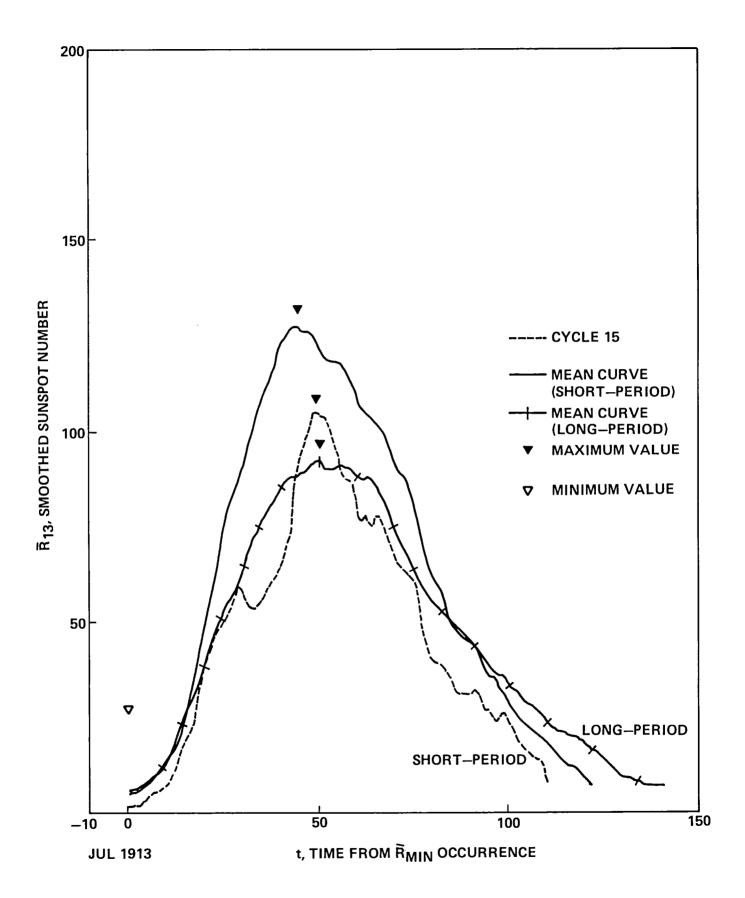


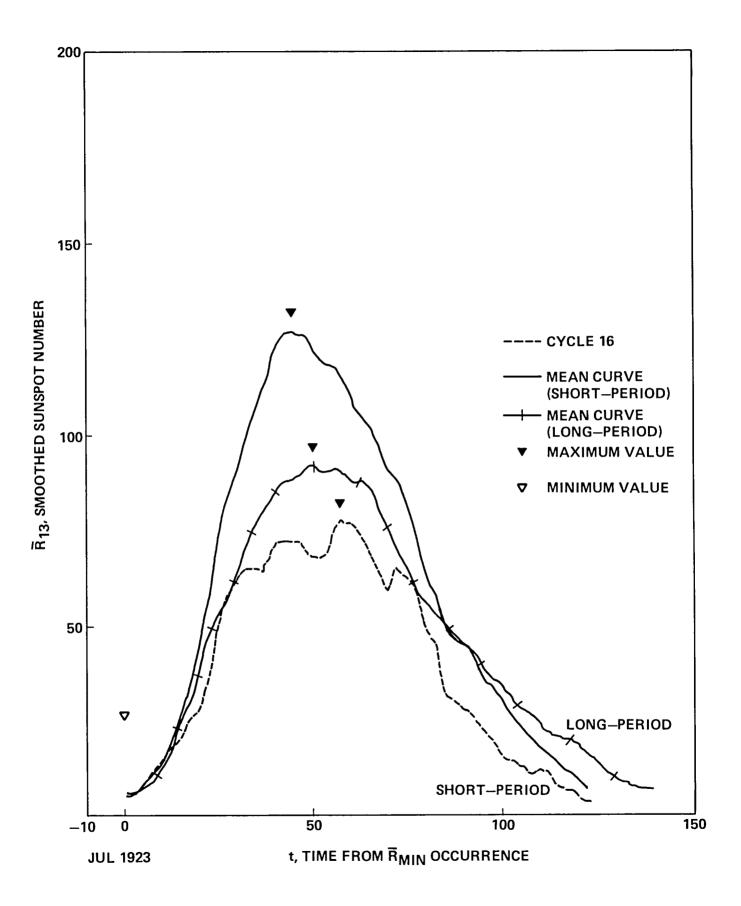


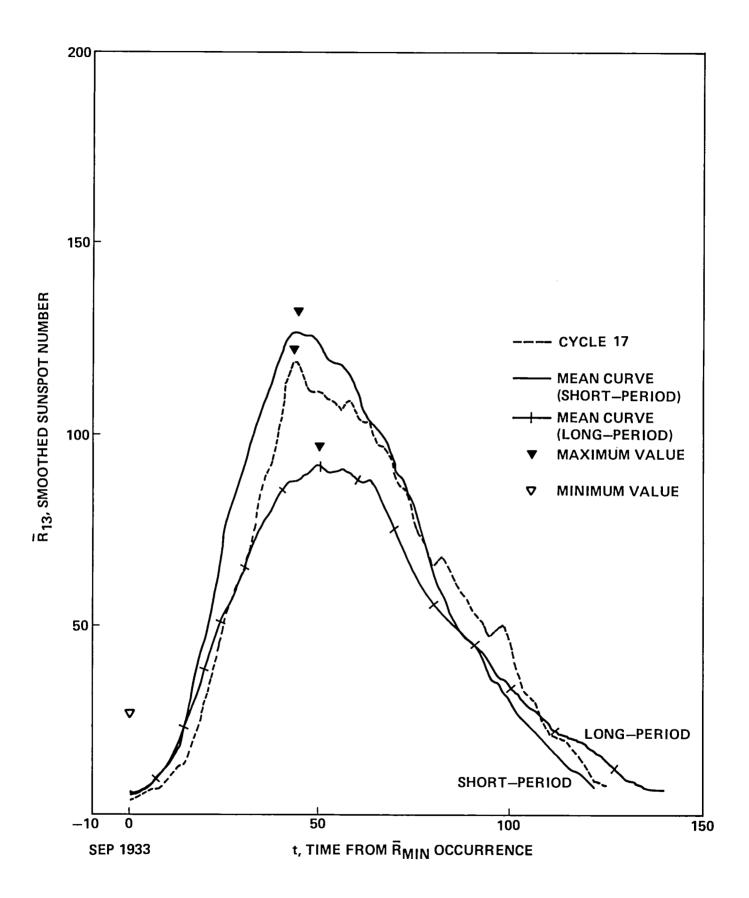


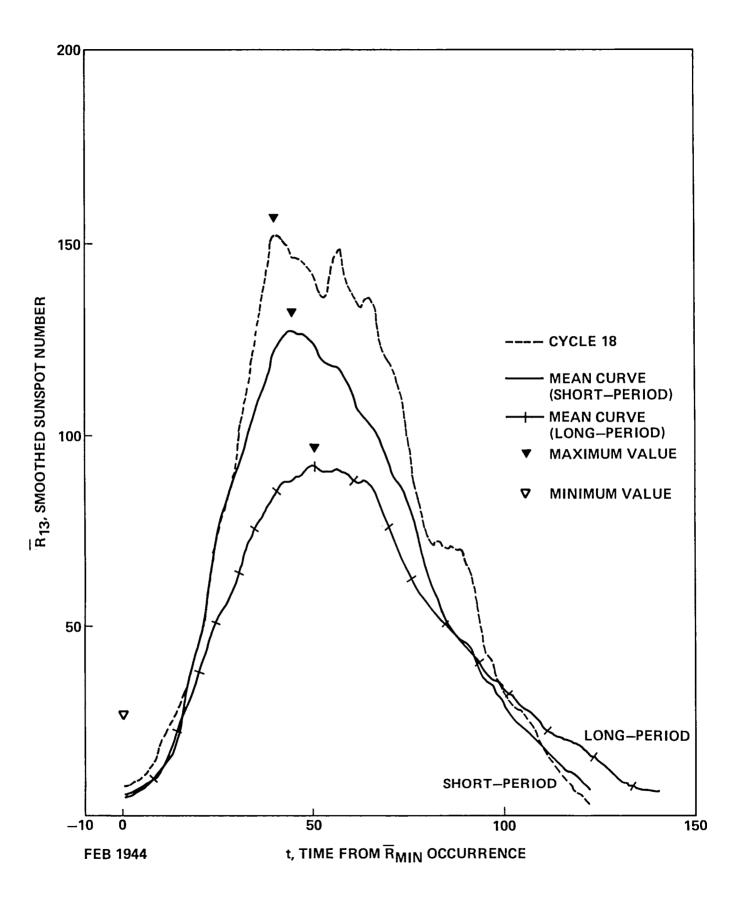


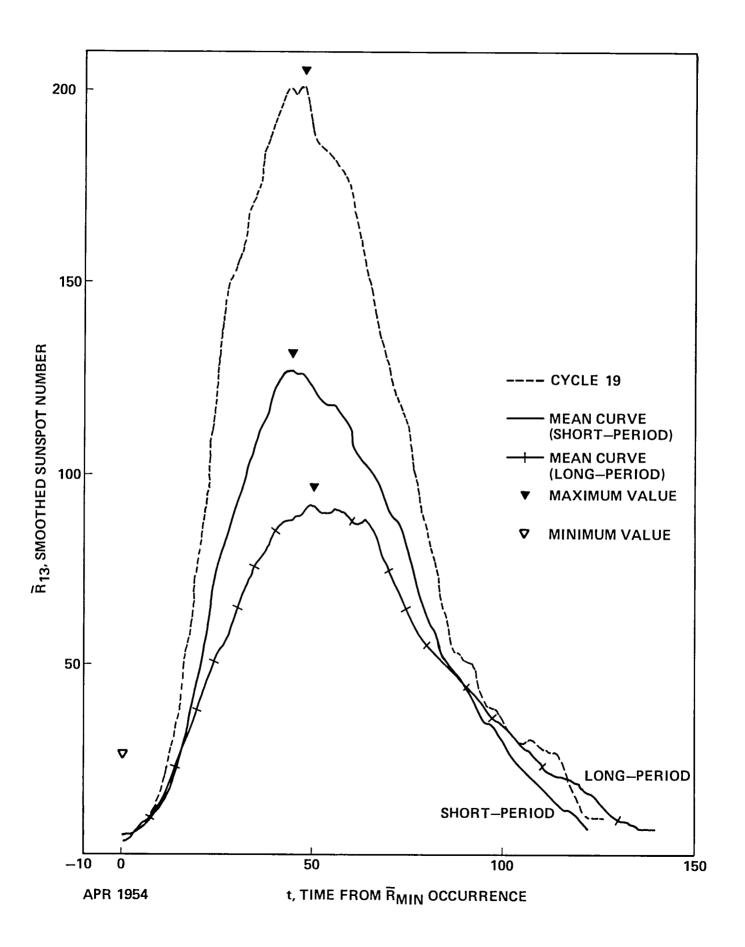


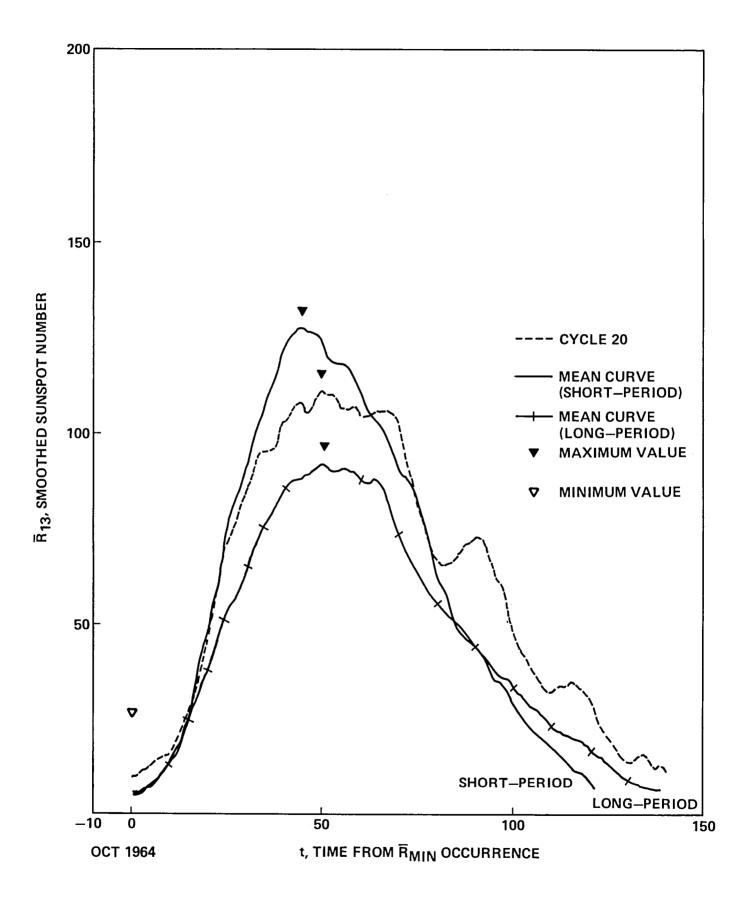












APPENDIX F

SCN	$ \begin{array}{c} 12 \\ \Sigma R_z \text{ (t)} \\ t = o \end{array} $	$ \begin{array}{c} 18 \\ \Sigma R_z \text{ (t)} \\ t = 0 \end{array} $	$ \begin{array}{c} 24 \\ \Sigma R_z \text{ (t)} \\ t = 0 \end{array} $
8	140.2	331.5	778.5
9	158.0	268.3	521.6
10	54.9	136.9	327.6
11	144.5	349.5	710.8
12	72.5	220.1	459.4
13	115.0	327.3	651.4
14	68.9	184.3	384.8
15	58.5	148.8	444.6
16	133.2	247.7	449.4
17	83.8	175.8	367.1
18	142.6	331.4	642.3
19	101.4	330.6	923.4
20	176.5	335.9	644.2
21	203.3	414.9	914.4
x	111.54	260.62	561.93
s	39.19	76.28	171.82
r	0.36	0.55	0.91
s_{yx}	37.17	33.28	16.70
a _{yx}	75.591	47.376	44.848
b _{yx}	0.337	0.264	0.134

1. REPORT NO.	2. GOVERNMENT ACC	ESSION NO.	3. RECIPIENT'S CA	ATALOG NO.	
NASA TP-2325					
4. TITLE AND SUBTITLE	· · · · · · · · · · · · · · · · · · ·		5. REPORT DATE		
A Comparative Look at Sunspot Cy	May 1984 6. PERFORMING ORGANIZATION CODE				
7. AUTHOR(S) Robert M. Wilson		8. PERFORMING ORGANIZATION REPORT			
9. PERFORMING ORGANIZATION NAME AND A	DDRESS		10. WORK UNIT NO.	·	
Coons C. Maraball Coos Eli 14 C	4		M-450		
George C. Marshall Space Flight Cen Marshall Space Flight Center, Alaba		11. CONTRACT OR G	RANT NO.		
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12. SPONSORING AGENCY NAME AND ADDRESS	S				
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Washington, D.C. 20546			14. SPONSORING AC	SENCY CODE	
15. SUPPLEMENTARY NOTES			<u> </u>		
Prepared by Space Science Laborato	ory, Science and En	gineering Directorat	e.		
16. ABSTRACT					
io. Application					
On the basis of cycles 8 thro	ough 20, spanning a	bout 143 years, obs	servations of suns	pot number,	
smoothed sunspot number, and thei	r temporal properti	es have been used to	o compute means	, standard	
deviations, ranges, and frequency of	occurrence histogra	ams for a number o	f sunspot cycle pa	arameters	
(e.g., \overline{R}_{MIN} , R_{MAX} , ASC, DES, etc	.). The resultant "s	chematic" sunspot o	cycle has been con	ntrasted with	
the "mean" sunspot cycle, obtained					
all cycles (8 through 20) to their mi	inimum occurrence	date. A relatively go	ood approximatio	n of the time	
variation of smoothed sunspot numb	oer for a given cycle	e is possible if sunsp	oot cycles are rega	arded in terms	
of being either HIGH- or LOW- \overline{R}_{MA}	X cycles or LONG-	or SHORT-PERIO	D cycles, especial	ly the latter.	
Further, sunspot cycles denoted HIC					
LOW-RMAX usually are LONG-PER					
ing late cycle parameters with early					
	24				
occurring cycle parameters \overline{R}_{MIN} , G	Δ \bar{R}_{13} and \sum R	7(t), especially the	latter two, can be	e used to	
miv G	t=0		,		
estimate later occurring cycle parameter	eters with relatively	good success, based	d on cycle 21 as a	an example.	
The sunspot cycle record clearly sho	ws that the trend f	or both \overline{R}_{MIN} and	\overline{R}_{MAX} was towar	rd decreasing	
value between cycles 8 through 14 a					
regression equations have also been of	obtained for several	measures of solar a	ctivity – R. R.	(now R ₁).	
\overline{R}_{13} , F_{2800} , and \overline{F}_{13} on the basis of	of provisional and fi	inal values.	, A, Z	1, (
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